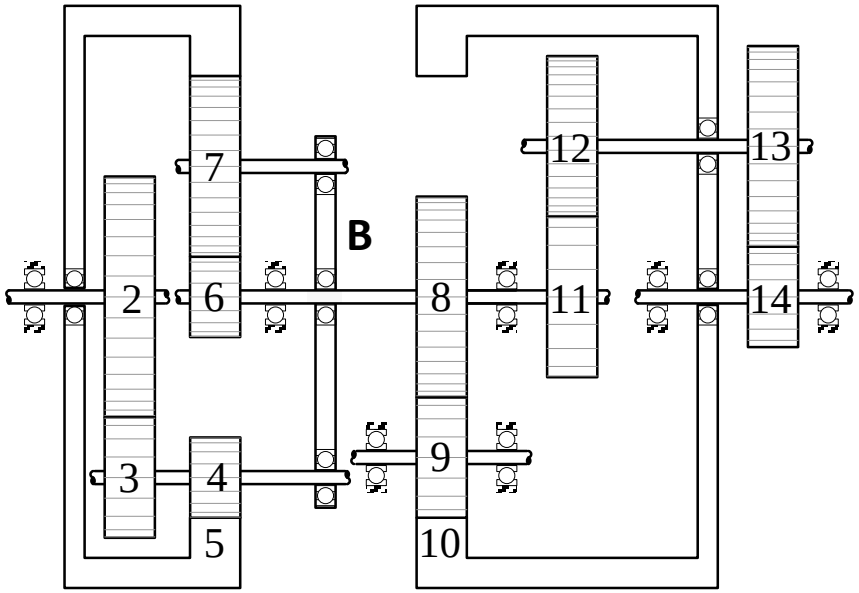
EJERCICIO 1

En la figura se muestra un tren de engranajes. Se pide plantear las ecuaciones que permiten obtener la velocidad de la rueda 14, considerando conocidas las velocidades de la rueda 2 y la corona 5



En negrita las incógnitas de cada ecuación

2 3 4 5 Brazo B

$$\frac{\omega_2 - \boldsymbol{\omega_B}}{\omega_5 - \omega_B} = -\frac{z_3\, z_5}{z_2 z_4} \Rightarrow \boxed{\boldsymbol{\omega_B}}$$

6 7 5 Brazo B

$$\frac{\boldsymbol{\omega_6} - \omega_B}{\omega_5 - \omega_B} = -\frac{z_5}{z_6} \Rightarrow \boxed{\boldsymbol{\omega_6 = \omega_8 = \omega_{11}}}$$

8 9 10 Ordinario

$$\frac{\omega_8}{\boldsymbol{\omega_{10}}} = -\frac{z_9\, z_{10}}{z_8\, z_9} \Rightarrow \boxed{\boldsymbol{\omega_{10}}}$$

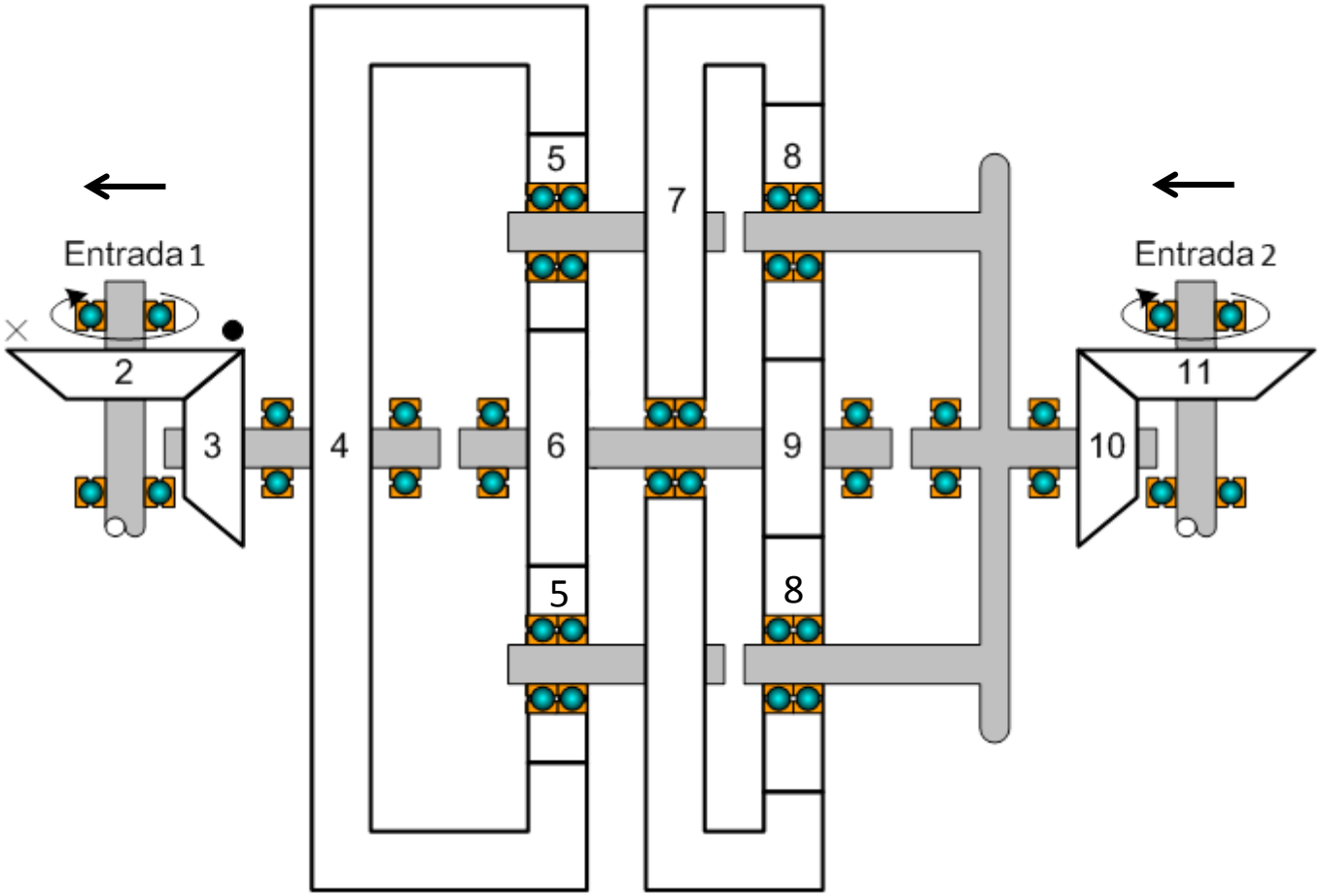
11 12 13 14 Brazo 10

$$\frac{\omega_{11} - \omega_{10}}{\boldsymbol{\omega_{14}} - \omega_{10}} = +\frac{z_{12}\, z_{14}}{z_{11} z_{13}} \Rightarrow \boxed{\boldsymbol{\omega_{14}}}$$

EJERCICIO 2

4- En la figura se representa el esquema de un tren de engranajes epicicloidal en el que las ruedas **2 y 11 giran ambas a 1000 rpm en sentido horario** visto desde arriba. En este tren la salida es el eje de las ruedas 6 y 9 que giran a la misma velocidad. Determinar la **velocidad y el sentido de giro de las rueda 6 y 9** *(el sentido de giro se dará **visto de la derecha**)*

DATOS: $Z_2 = Z_3 = Z_5 = Z_{10} = Z_{11} = Z_9 = 15$; $Z_8 = Z_6 = 30$; $Z_7 = 75$; $Z_4 = 60$



23 Ordinario

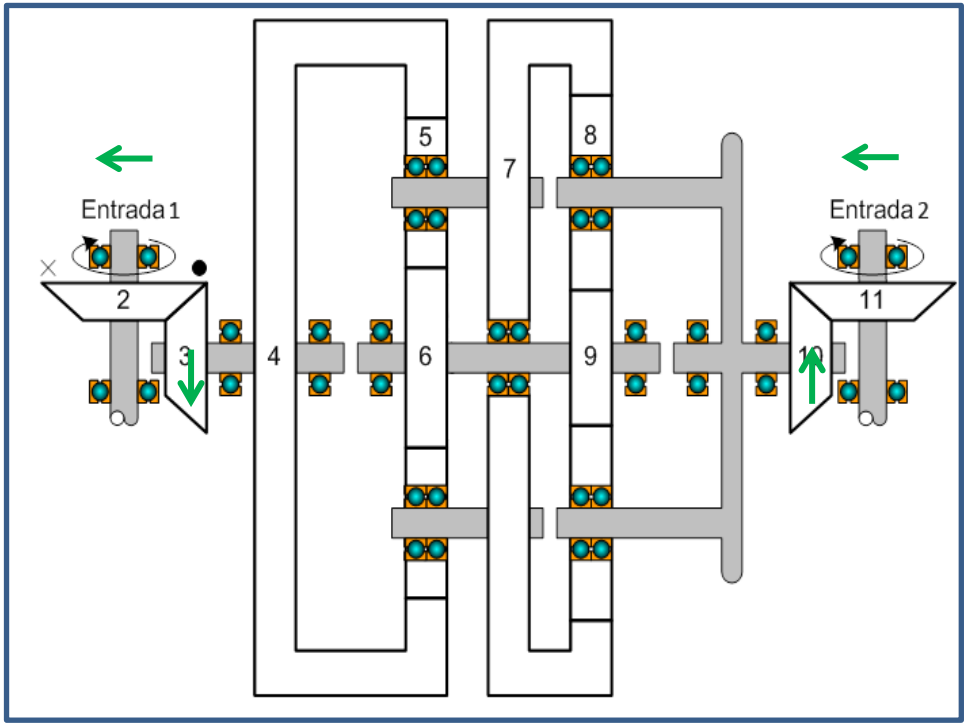
$$\frac{\omega_2}{\boldsymbol{\omega_3}} = \frac{z_3}{z_2} = 1 \Rightarrow \boxed{\omega_3 = 1000\ (sa) = \omega_4}$$

$$(sa) \rightarrow positivo$$

10 11 Ordinario

$$\frac{\omega_{11}}{\boldsymbol{\omega_{10}}} = \frac{z_{10}}{z_{11}} = 1 \Rightarrow \boxed{\omega_{10} = -1000}$$

$$(sh)$$



23 Ordinario

$$\frac{\omega_2}{\boldsymbol{\omega_3}} = \frac{z_3}{z_2} = 1 \quad \Rightarrow \quad \omega_3 = 1000\ (sa) = \omega_4$$

$$(sa) \rightarrow positivo$$

10 11 Ordinario

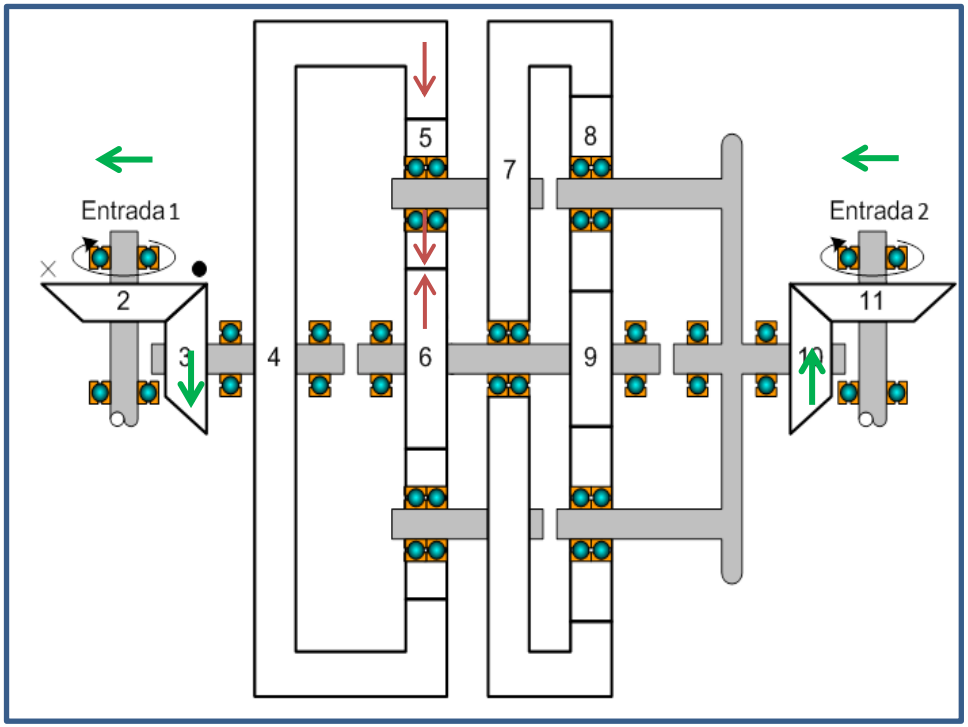
$$\frac{\omega_{11}}{\boldsymbol{\omega_{10}}} = \frac{z_{10}}{z_{11}} = 1 \quad \Rightarrow \quad \omega_{10} = -1000$$

$$(sh)$$

6 5 4 Brazo 7

$$\frac{\boldsymbol{\omega_6} - \boldsymbol{\omega_7}}{\omega_4 - \boldsymbol{\omega_7}} = -\frac{z_4}{z_6} = -2$$

$$\frac{\boldsymbol{\omega_6} - \boldsymbol{\omega_7}}{1000 - \boldsymbol{\omega_7}} = -2 \quad \Rightarrow \quad \boldsymbol{\omega_7} = \frac{\boldsymbol{\omega_6} + 2000}{3}$$



23 Ordinario

$$\frac{\omega_2}{\boldsymbol{\omega_3}} = \frac{z_3}{z_2} = 1 \quad \Rightarrow \quad \omega_3 = 1000\ (sa) = \omega_4$$

$$(sa) \rightarrow positivo$$

10 11 Ordinario

$$\frac{\omega_{11}}{\boldsymbol{\omega_{10}}} = \frac{z_{10}}{z_{11}} = 1 \quad \Rightarrow \quad \omega_{10} = -1000$$

$$(sh)$$

6 5 4 Brazo 7

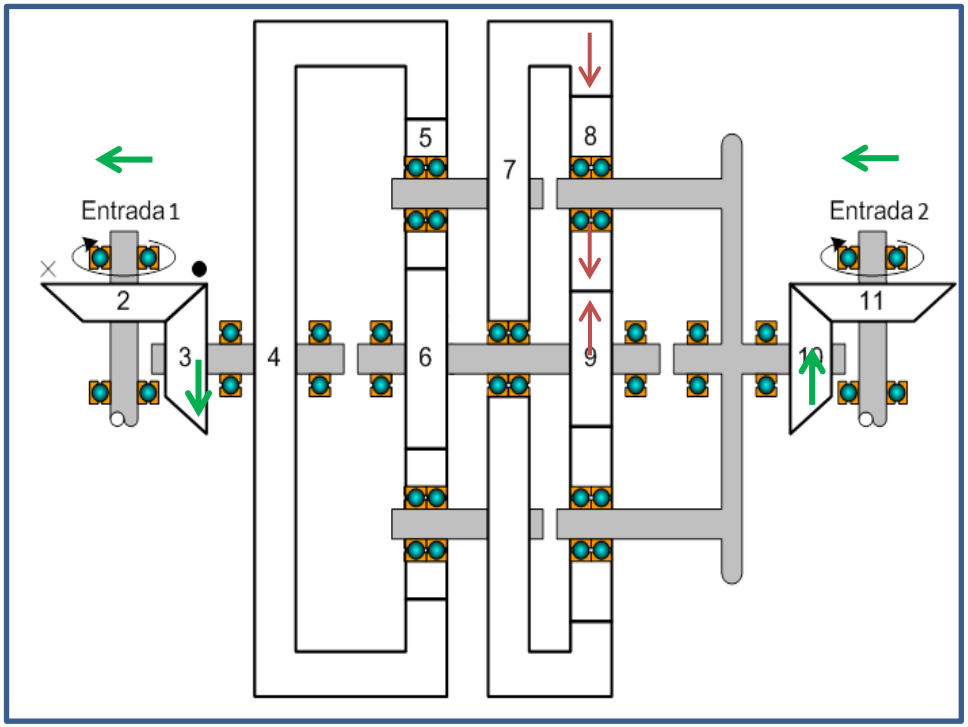
$$\frac{\boldsymbol{\omega_6} - \boldsymbol{\omega_7}}{\omega_4 - \boldsymbol{\omega_7}} = -\frac{z_4}{z_6} = -2$$

$$\frac{\boldsymbol{\omega_6} - \boldsymbol{\omega_7}}{1000 - \boldsymbol{\omega_7}} = -2 \quad \Rightarrow \quad \boldsymbol{\omega_7} = \frac{\boldsymbol{\omega_6} + 2000}{3}$$

9 8 7 Brazo 10

$$\frac{\boldsymbol{\omega_9} - \omega_{10}}{\boldsymbol{\omega_7} - \omega_{10}} = -\frac{z_7}{z_9} = -5$$

$$\frac{\boldsymbol{\omega_9} - (-1000)}{\boldsymbol{\omega_7} - (-1000)} = -5$$



23 Ordinario

$$\frac{\omega_2}{\omega_3} = \frac{z_3}{z_2} = 1 \Rightarrow \boxed{\omega_3 = 1000\ (sa) = \omega_4}$$

$$(sa) \rightarrow positivo$$

10 11 Ordinario

$$\frac{\omega_{11}}{\omega_{10}} = \frac{z_{10}}{z_{11}} = 1 \Rightarrow \boxed{\omega_{10} = -1000}$$

$$(sh)$$

6 5 4 Brazo 7

$$\frac{\omega_6 - \omega_7}{\omega_4 - \omega_7} = -\frac{z_4}{z_6} = -2$$

$$\frac{\omega_6 - \omega_7}{1000 - \omega_7} = -2 \Rightarrow \omega_7 = \frac{\omega_6 + 2000}{3}$$

9 8 7 Brazo 10

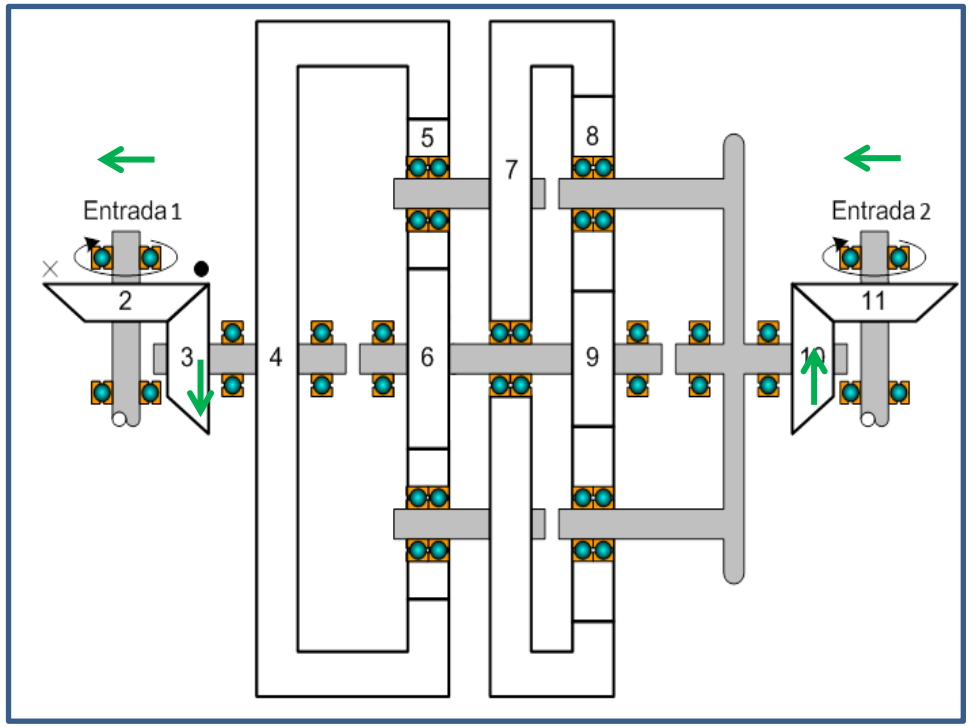
$$\frac{\omega_9 - \omega_{10}}{\omega_7 - \omega_{10}} = -\frac{z_7}{z_9} = -5$$

$$\frac{\omega_9 - (-1000)}{\omega_7 - (-1000)} = -5 \Rightarrow \omega_7 = -\frac{\omega_9 + 6000}{5}$$

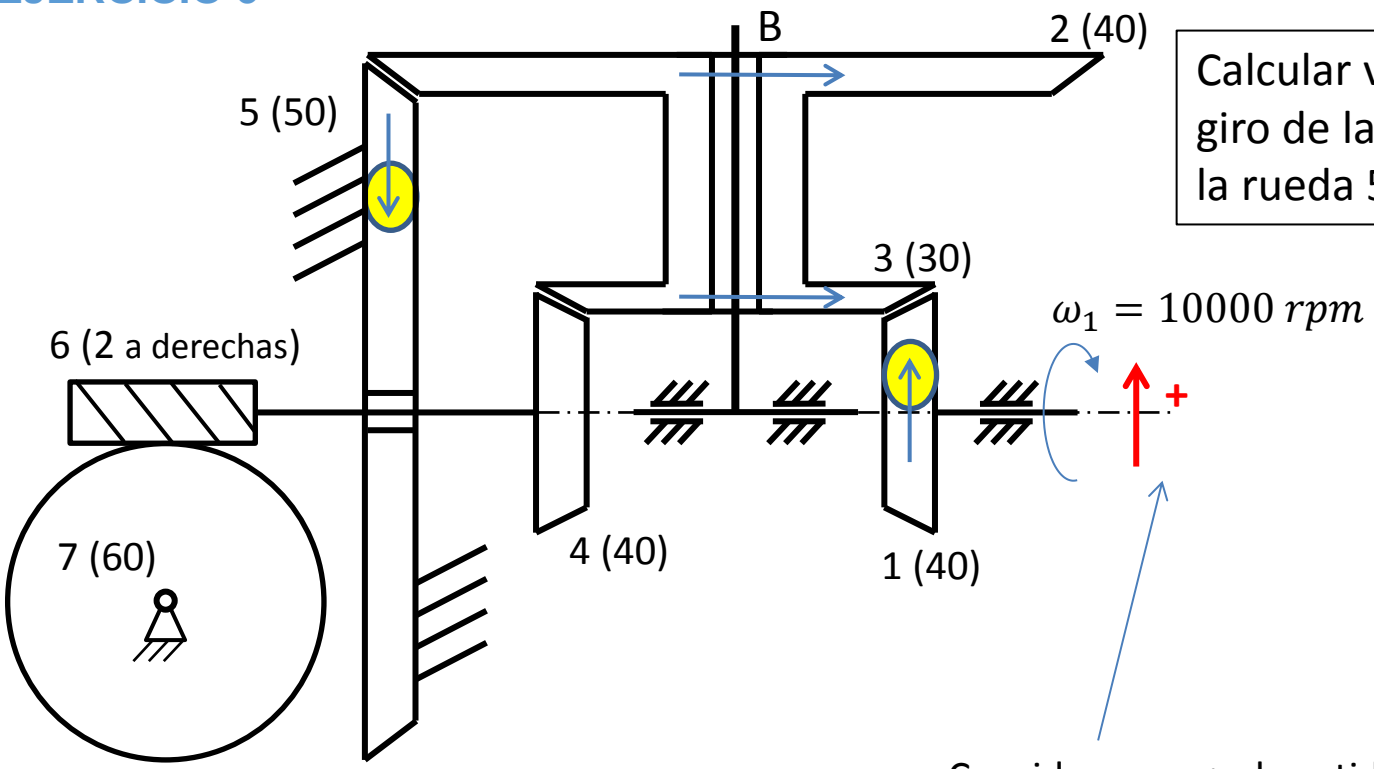
$$(\omega_6 = \omega_9) \rightarrow \frac{\omega_9 + 2000}{3} = -\frac{\omega_9 + 6000}{5}$$

$$\boxed{\omega_6 = \omega_9 = -3500 \text{ rpm}}$$

$$(sh)$$



EJERCICIO 3

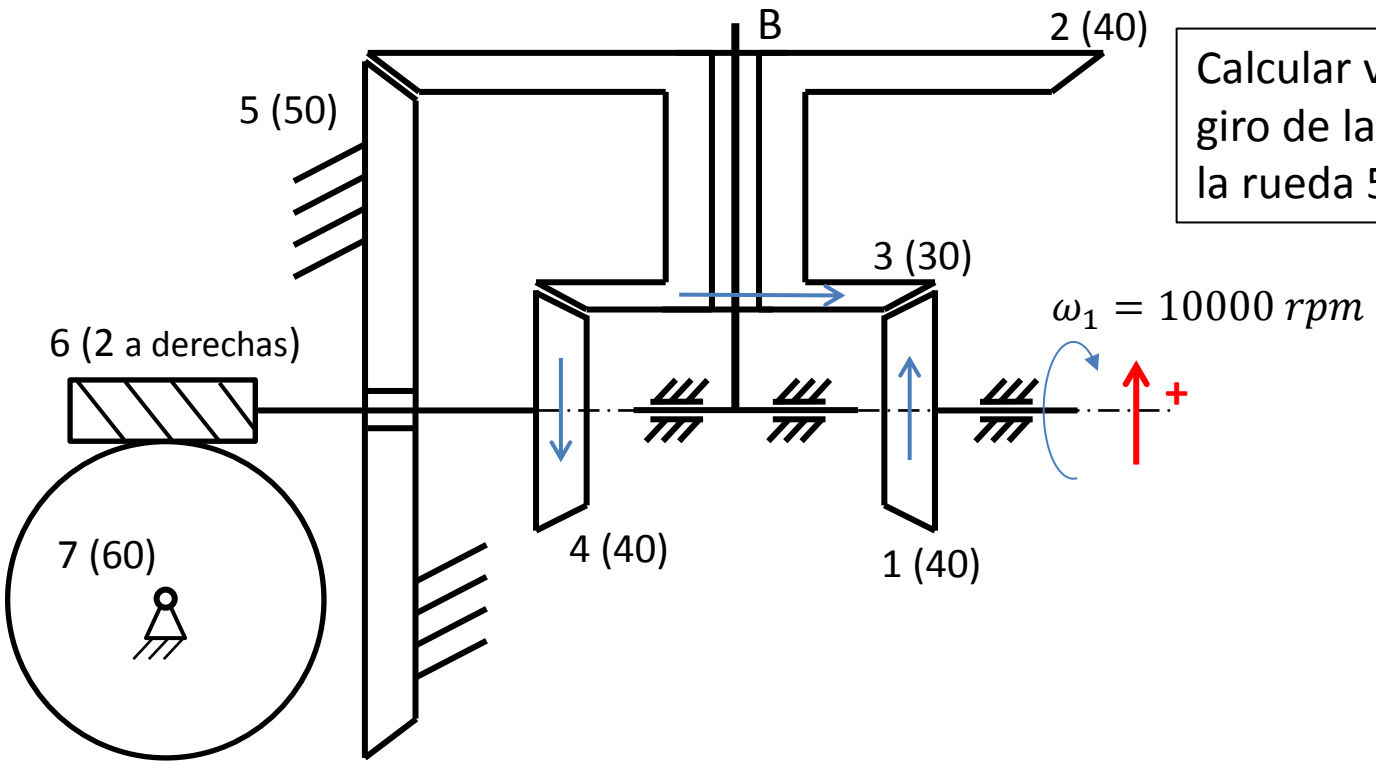


Calcular velocidad y sentido de giro de la rueda 7 cuando se fija la rueda 5 y 1 gira a 10000 rpm

Consideramos + el sentido de w1

1º) 1 3 2 5 Brazo B

$$\frac{\omega_1 - \omega_B}{\omega_5 - \omega_B} = -\frac{z_3\, z_5}{z_1 z_2} = -\frac{30\ 50}{40\ 40} = -0{,}9375 = \frac{10000 - \omega_B}{0 - \omega_B} \longrightarrow \boxed{\omega_B = 5161{,}29}$$



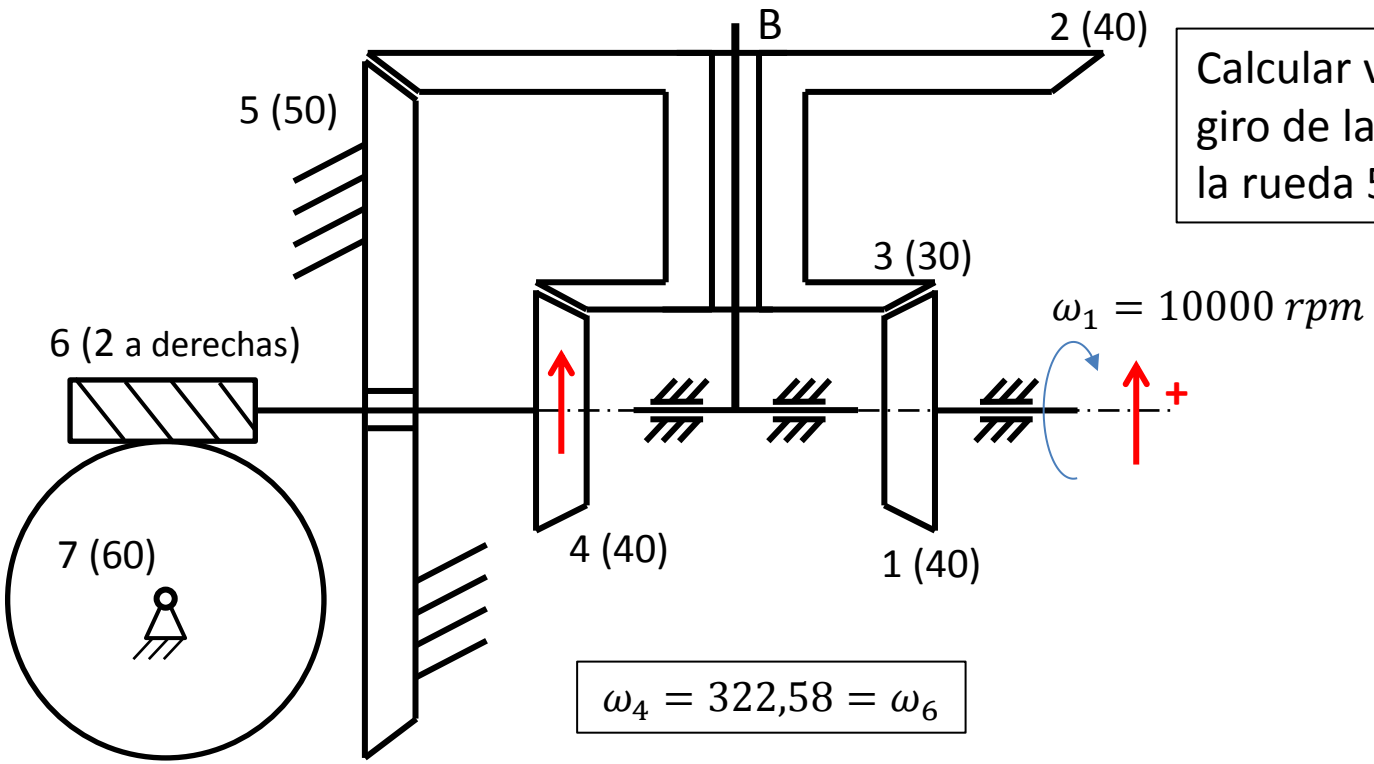
Calcular velocidad y sentido de giro de la rueda 7 cuando se fija la rueda 5 y 1 gira a 10000 rpm

1º) 1 3 2 5 Brazo B

$$\frac{\omega_1 - \omega_B}{\omega_5 - \omega_B} = -\frac{z_3\, z_5}{z_1 z_2} = -\frac{30\ 50}{40\ 40} = -0{,}9375 = \frac{10000 - \omega_B}{0 - \omega_B} \longrightarrow \boxed{\omega_B = 5161{,}29}$$

2º) 1 3 4 Brazo B

$$\frac{\omega_1 - \omega_B}{\omega_4 - \omega_B} = -\frac{z_3\, z_4}{z_1 z_3} = -\frac{40}{40} = -1 = \frac{10000 - 5161{,}29}{\omega_4 - 5161{,}29} \longrightarrow \boxed{\omega_4 = 322{,}58 = \omega_6}$$

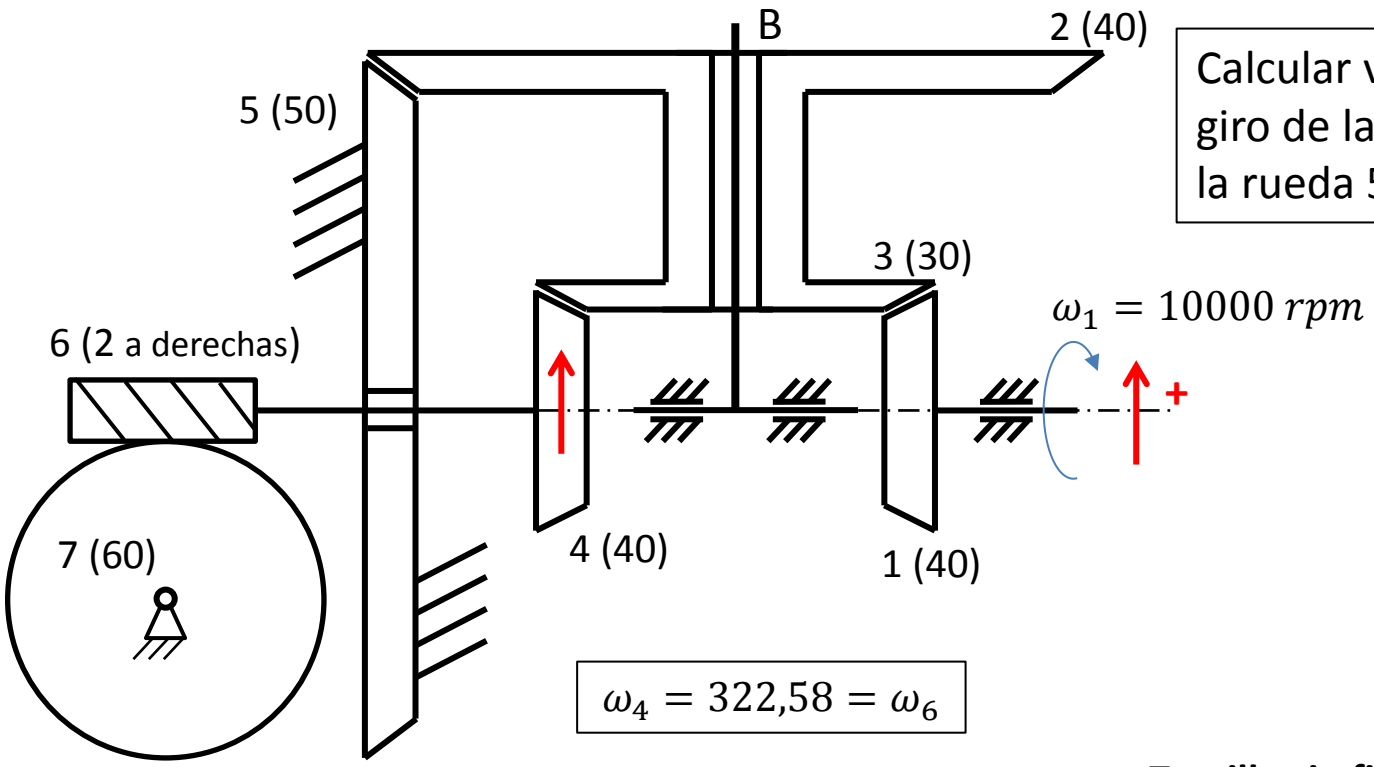


Calcular velocidad y sentido de giro de la rueda 7 cuando se fija la rueda 5 y 1 gira a 10000 rpm

$\omega_4 = 322{,}58 = \omega_6$

1º) 6 7 Ordinario

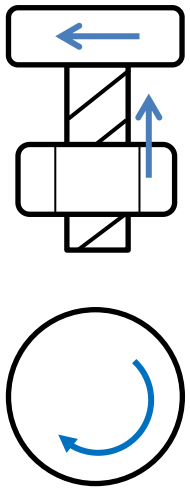
$$\frac{\omega_6}{\omega_7} = \frac{z_7}{z_6} = \frac{60}{2} = 30 \rightarrow \omega_7 = 10.75\ rpm$$



Calcular velocidad y sentido de giro de la rueda 7 cuando se fija la rueda 5 y 1 gira a 10000 rpm

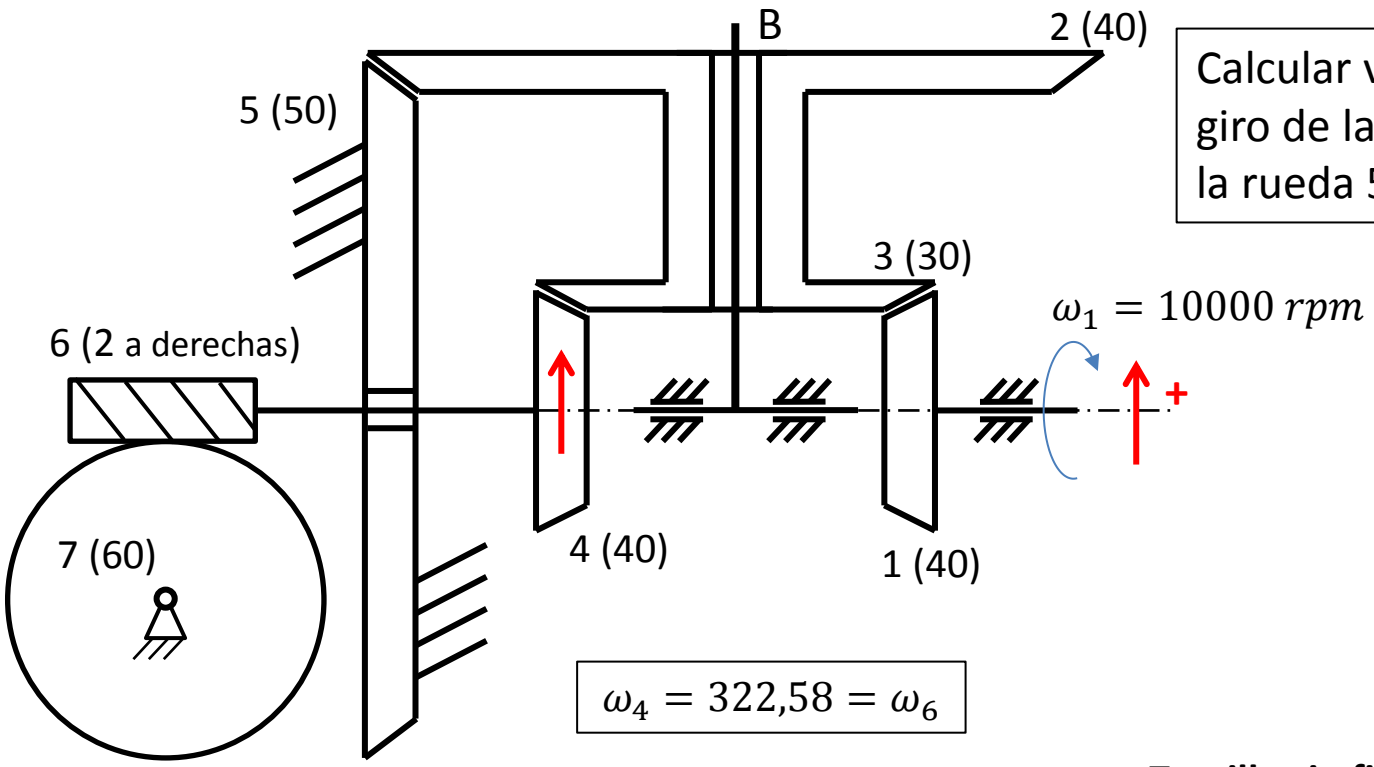
$$\omega_4 = 322{,}58 = \omega_6$$

Tornillo sin fin rosca a derechas:



1º) 6 7 Ordinario

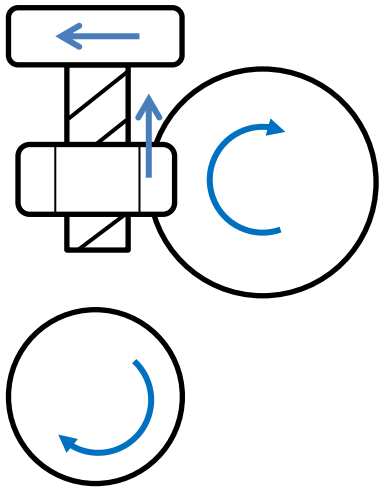
$$\frac{\omega_6}{\omega_7} = \frac{z_7}{z_6} = \frac{60}{2} = 30 \rightarrow \omega_7 = 10.75\ rpm$$



Calcular velocidad y sentido de giro de la rueda 7 cuando se fija la rueda 5 y 1 gira a 10000 rpm

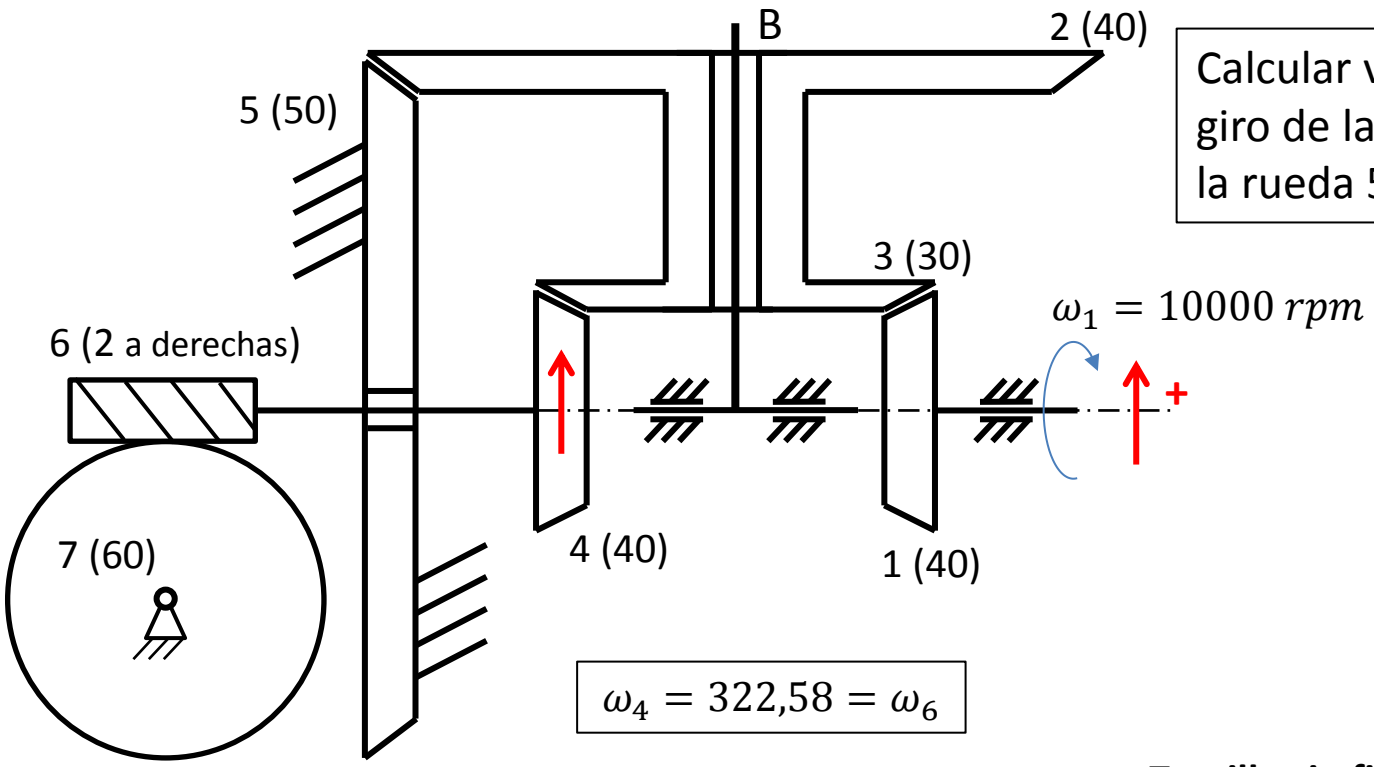
$$\omega_4 = 322{,}58 = \omega_6$$

Tornillo sin fin rosca a derechas:



1º) 6 7 Ordinario

$$\frac{\omega_6}{\omega_7} = \frac{z_7}{z_6} = \frac{60}{2} = 30 \rightarrow \omega_7 = 10.75\ rpm$$



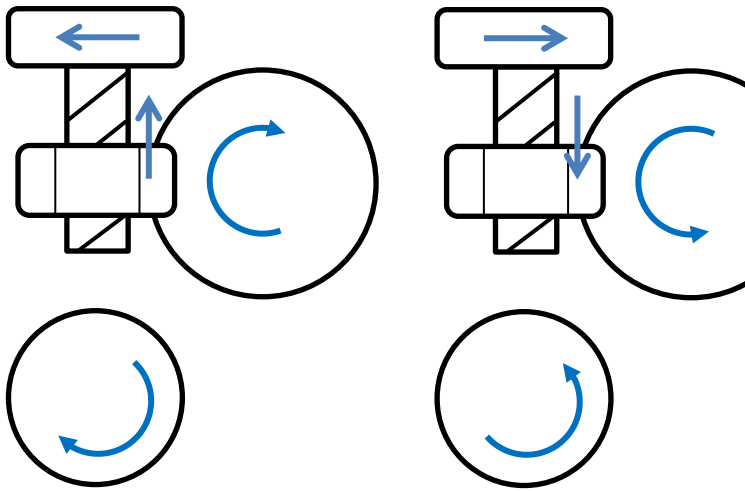
Calcular velocidad y sentido de giro de la rueda 7 cuando se fija la rueda 5 y 1 gira a 10000 rpm

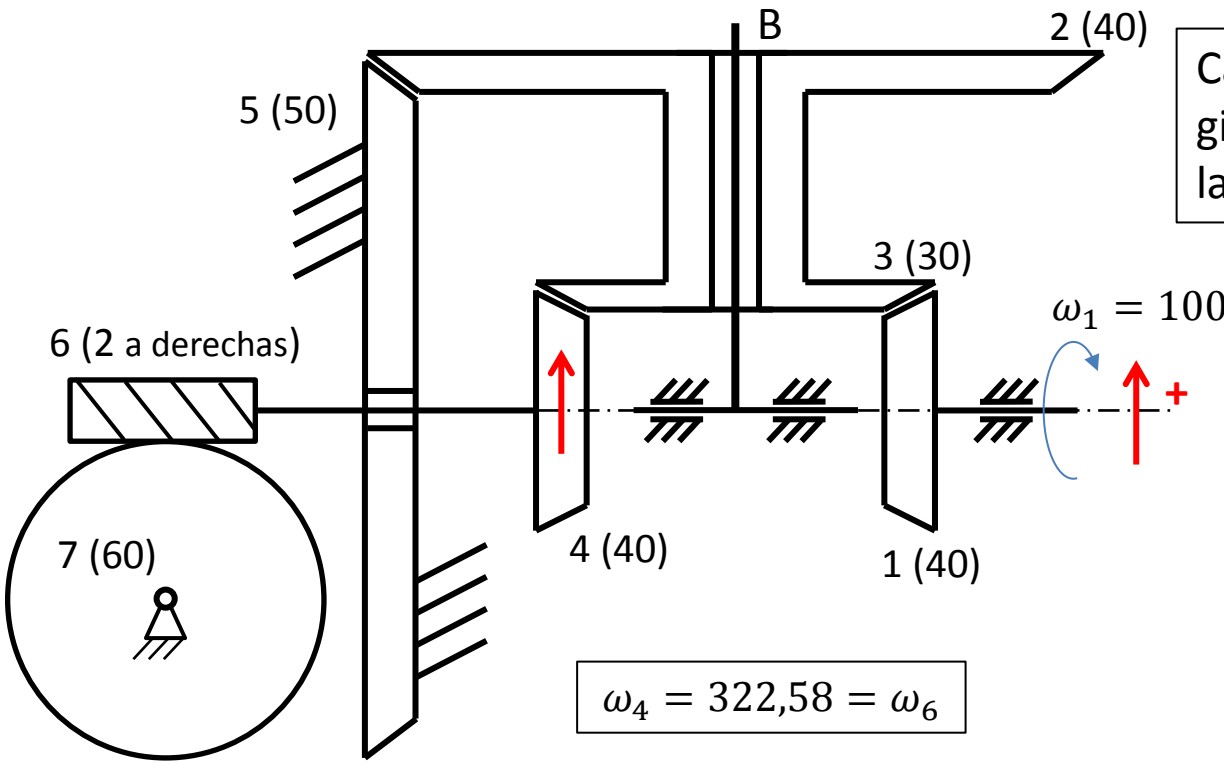
$\omega_4 = 322{,}58 = \omega_6$

1º) 6 7 Ordinario

$$\frac{\omega_6}{\omega_7} = \frac{z_7}{z_6} = \frac{60}{2} = 30 \rightarrow \omega_7 = 10.75\ rpm$$

Tornillo sin fin rosca a derechas:

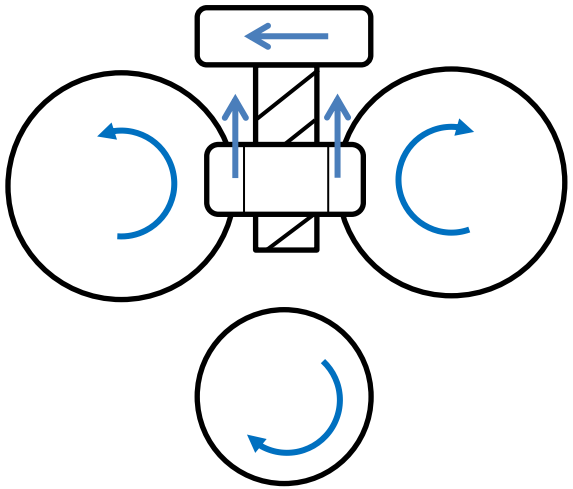




Calcular velocidad y sentido de giro de la rueda 7 cuando se fija la rueda 5 y 1 gira a 10000 rpm

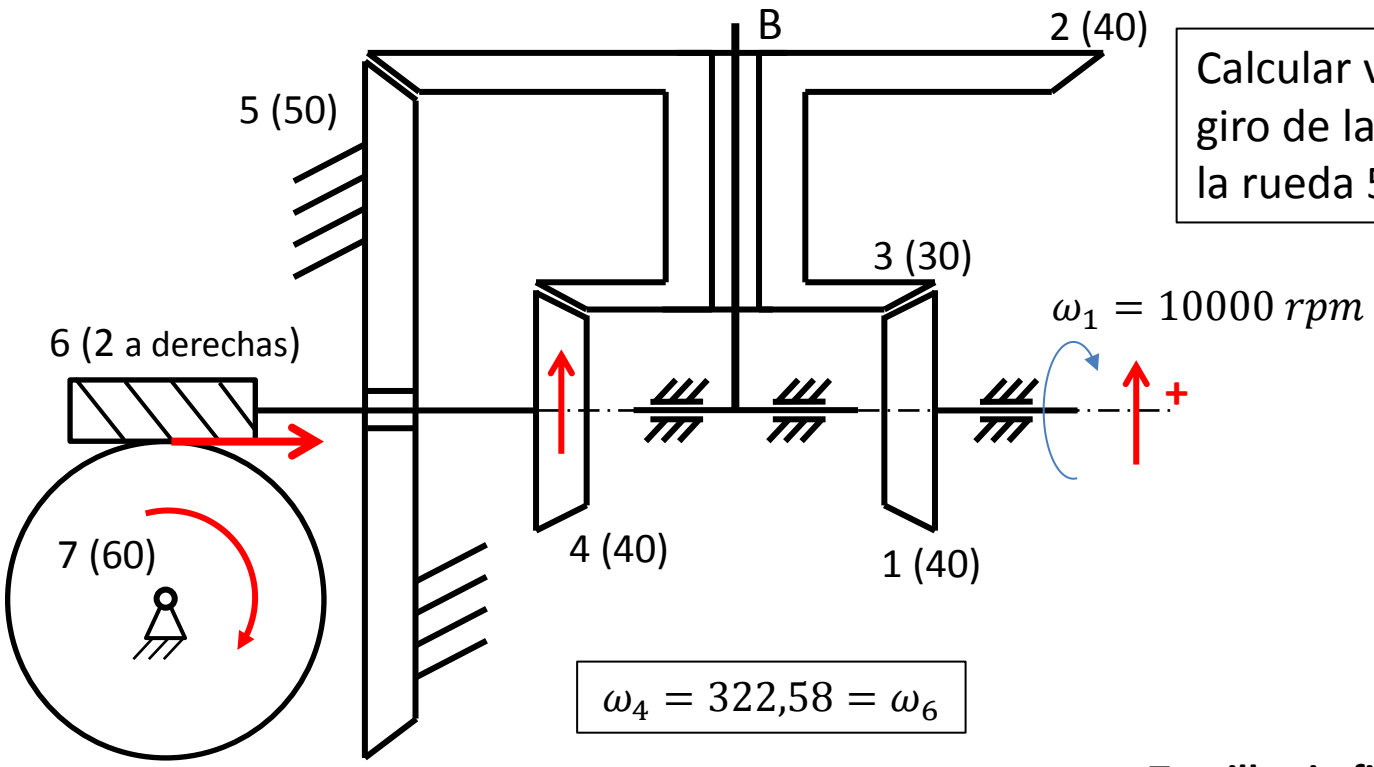
$$\omega_4 = 322{,}58 = \omega_6$$

Tornillo sin fin rosca a derechas:



1º) 6 7 Ordinario

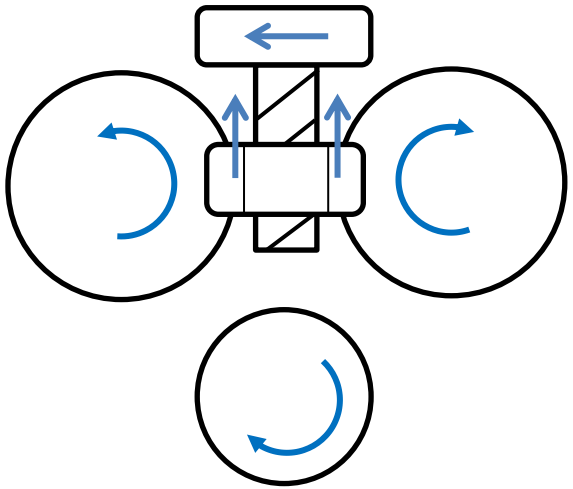
$$\frac{\omega_6}{\omega_7} = \frac{z_7}{z_6} = \frac{60}{2} = 30 \rightarrow \omega_7 = 10.75\ rpm$$



Calcular velocidad y sentido de giro de la rueda 7 cuando se fija la rueda 5 y 1 gira a 10000 rpm

$\omega_4 = 322{,}58 = \omega_6$

Tornillo sin fin rosca a derechas:



1º) 6 7 Ordinario

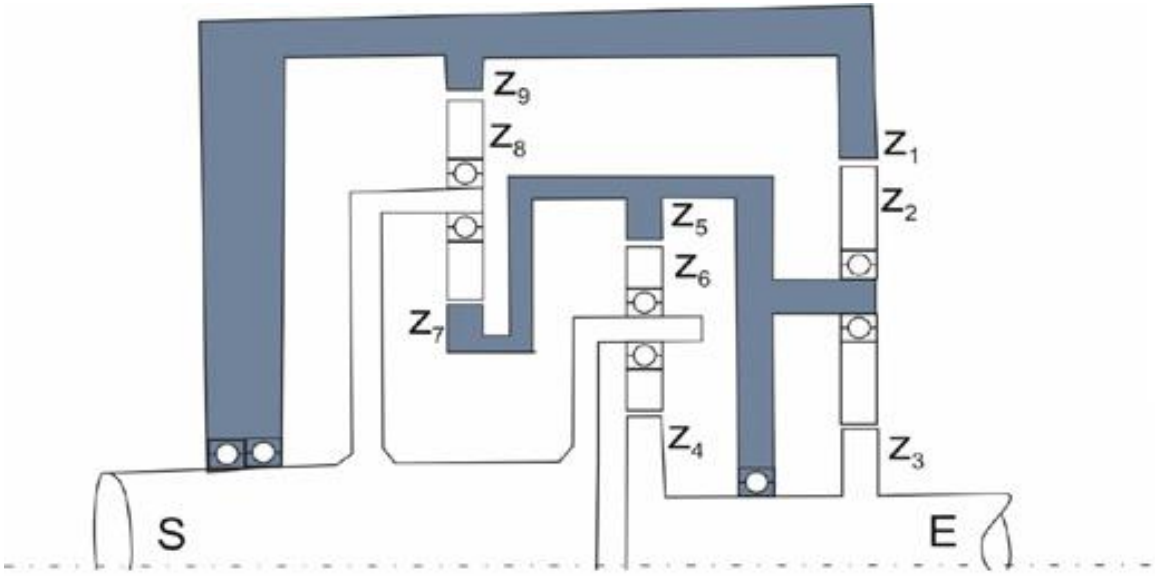
$$\frac{\omega_6}{\omega_7} = \frac{z_7}{z_6} = \frac{60}{2} = 30$$

$\boldsymbol{\omega_7 = 10.75\ rpm}$
En sentido horario

EJERCICIO 4

El esquema de la figura representa un tren de engranajes epicicloidal de 3 etapas, donde E es el eje de entrada y S el de salida. Las ruedas 3 y 4 giran solidarias con el eje de entrada, mientras que los brazos de las otras 2 etapas epicicloidales son solidarios con el eje de salida. Los sólidos sombreados giran respecto al eje de salida (con engranajes 1 y 9) y al eje de entrada (con engranajes 5 y 7), respectivamente.

Encontrar la velocidad del eje de salida (modulo y sentido) cuando el eje de entrada gira a 1000 rpm en sentido horario.



$z_1 = z_5 = 120$

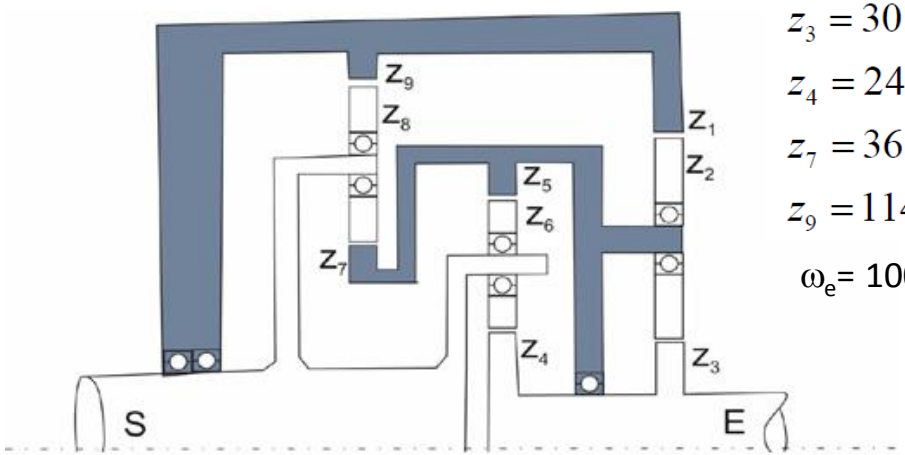
$z_3 = 30$

$z_4 = 24$

$z_7 = 36$

$z_9 = 114$

ω_e= 1000 rpm (sh)



$z_1 = z_5 = 120$

$z_3 = 30$

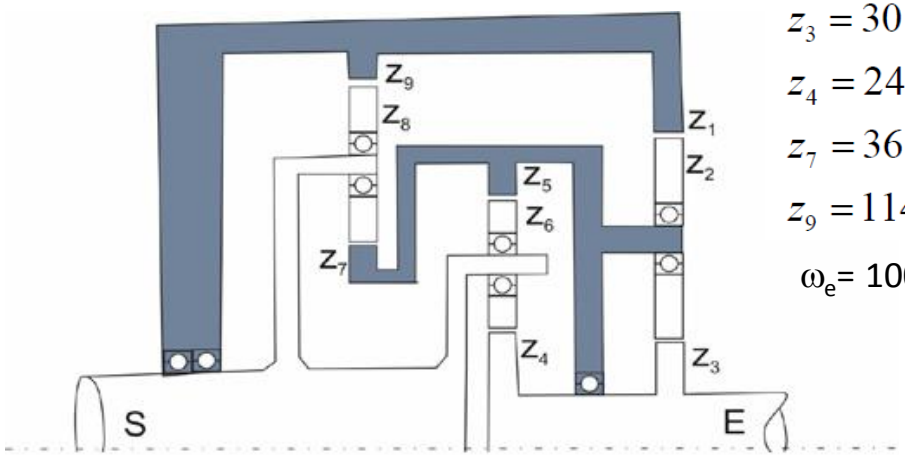
$z_4 = 24$

$z_7 = 36$

$z_9 = 114$

ω_e= 1000 rpm (sh)

1) 3 2 1 Brazo 5
2) 4 6 5 Brazo S
3) 7 8 9 Brazo S



$z_1 = z_5 = 120$

$z_3 = 30$

$z_4 = 24$

$z_7 = 36$

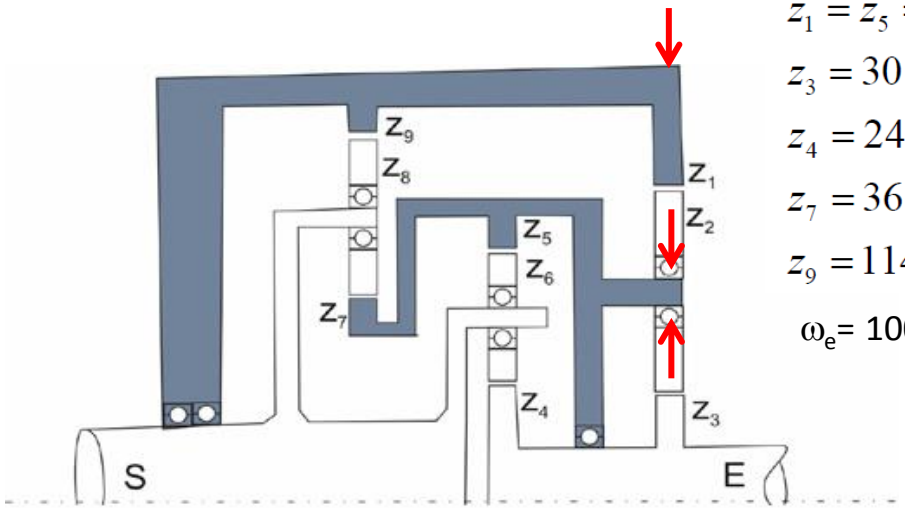
$z_9 = 114$

ω_e= 1000 rpm (sh)

1) 3 2 1 Brazo 5
2) 4 6 5 Brazo S
3) 7 8 9 Brazo S

1) 3 2 1 Brazo 5

$$\frac{\omega_3 - \omega_5}{\omega_1 - \omega_5} = \pm \frac{z_2\, z_1}{z_3 z_2}$$



$z_1 = z_5 = 120$

$z_3 = 30$

$z_4 = 24$

$z_7 = 36$

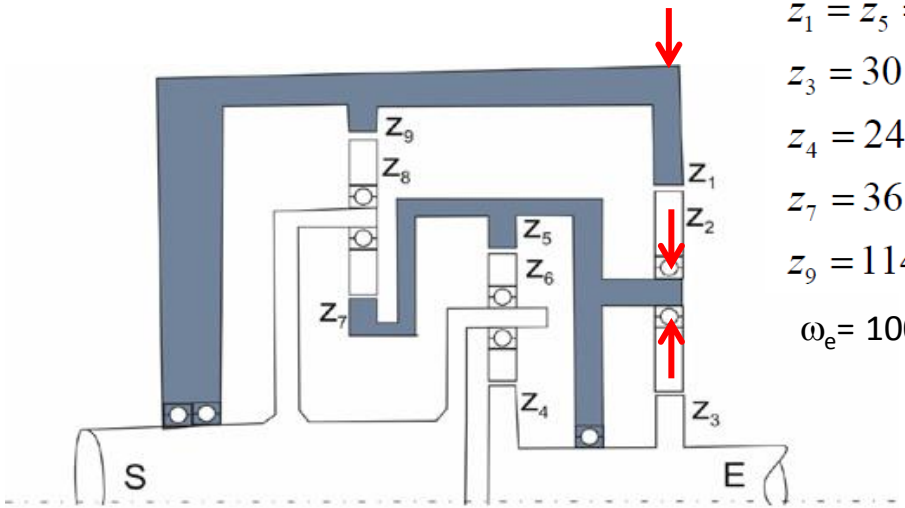
$z_9 = 114$

ω_e= 1000 rpm (sh)

1) 3 2 1 Brazo 5
2) 4 6 5 Brazo S
3) 7 8 9 Brazo S

1) 3 2 1 Brazo 5

$$\frac{\omega_3 - \omega_5}{\omega_1 - \omega_5} = -\frac{z_2\, z_1}{z_3 z_2} = -\frac{120}{30} = -4$$



$z_1 = z_5 = 120$

$z_3 = 30$

$z_4 = 24$

$z_7 = 36$

$z_9 = 114$

ω_e= 1000 rpm (sh)

1) 3 2 1 Brazo 5
2) 4 6 5 Brazo S
3) 7 8 9 Brazo S

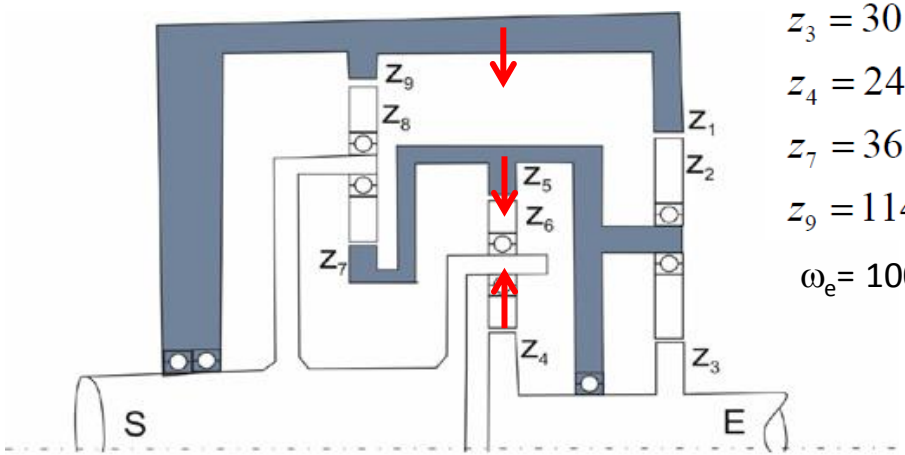
1) 3 2 1 Brazo 5

+

$$\frac{\omega_3 - \omega_5}{\omega_1 - \omega_5} = -\frac{z_2\, z_1}{z_3 z_2} = -\frac{120}{30} = -4$$

ω_3= ω_e = 1000 rpm (sh)

$$\frac{1000 - \omega_5}{\omega_1 - \omega_5} = -4 \rightarrow \boxed{\omega_5 = \frac{4\omega_1 + 1000}{5}}$$



$z_1 = z_5 = 120$

$z_3 = 30$

$z_4 = 24$

$z_7 = 36$

$z_9 = 114$

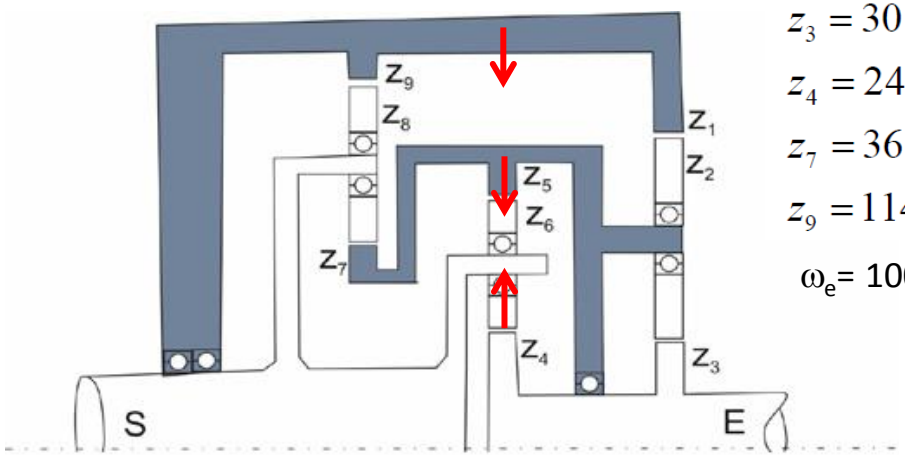
ω_e= 1000 rpm (sh) +

$$\omega_5 = \frac{4\omega_1 + 1000}{5}$$

1) 3 2 1 Brazo 5
2) 4 6 5 Brazo S
3) 7 8 9 Brazo S

2) 4 6 5 Brazo S

$$\frac{\omega_4 - \omega_s}{\omega_5 - \omega_s} = -\frac{z_6\, z_5}{z_4 z_6} = -\frac{120}{24} = -5$$



$z_1 = z_5 = 120$

$z_3 = 30$

$z_4 = 24$

$z_7 = 36$

$z_9 = 114$

ω_e= 1000 rpm (sh) +

$$\omega_5 = \frac{4\omega_1 + 1000}{5}$$

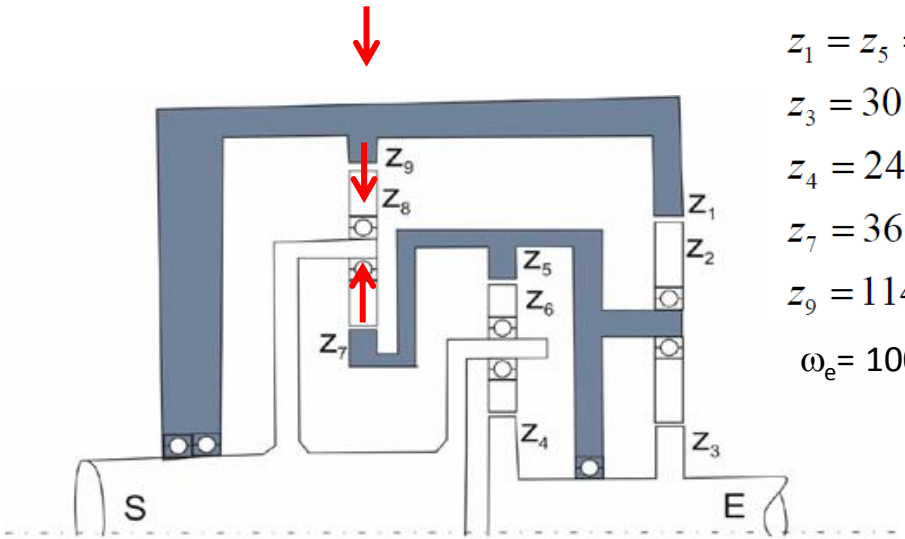
1) 3 2 1 Brazo 5
2) 4 6 5 Brazo S
3) 7 8 9 Brazo S

2) 4 6 5 Brazo S

$$\frac{\omega_4 - \omega_s}{\omega_5 - \omega_s} = -\frac{z_6\, z_5}{z_4 z_6} = -\frac{120}{24} = -5$$

$\omega_4 = \omega_e$= 1000 rpm (sh)

$$\frac{1000 - \omega_s}{\omega_5 - \omega_S} = -5 \longrightarrow \omega_5 = \frac{5\omega_s - 1000}{5}$$



$z_1 = z_5 = 120$

$z_3 = 30$

$z_4 = 24$

$z_7 = 36$

$z_9 = 114$

ω_e= 1000 rpm (sh) +

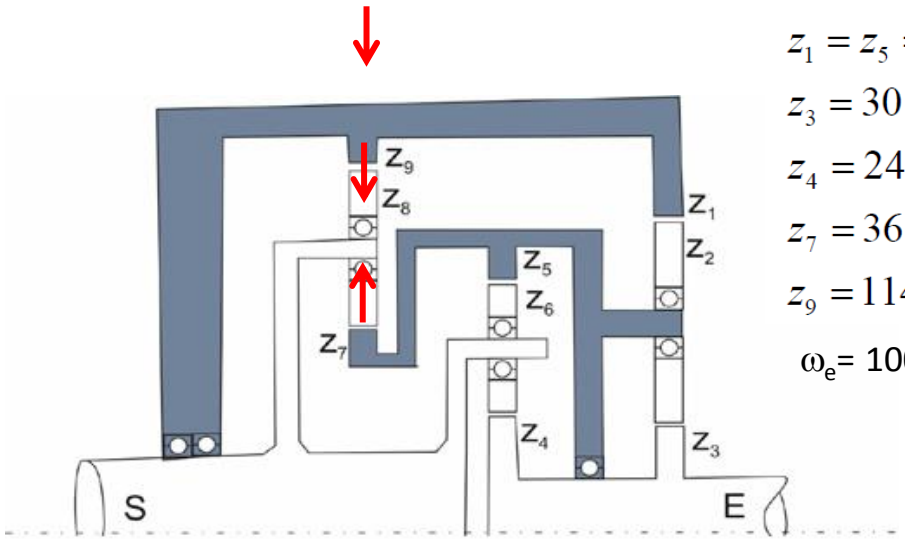
$$\omega_5 = \frac{4\omega_1 + 1000}{5}$$

$$\omega_5 = \frac{5\omega_s - 1000}{5}$$

1) 3 2 1 Brazo 5
2) 4 6 5 Brazo S
3) 7 8 9 Brazo S

3) 7 8 9 Brazo S

$$\frac{\omega_7 - \omega_s}{\omega_9 - \omega_s} = -\frac{z_8\, z_9}{z_7 z_8} = -\frac{114}{36} = -\frac{19}{6}$$



$z_1 = z_5 = 120$

$z_3 = 30$

$z_4 = 24$

$z_7 = 36$

$z_9 = 114$

ω_e= 1000 rpm (sh) [+]

$$\omega_5 = \frac{4\omega_1 + 1000}{5}$$

$$\omega_5 = \frac{5\omega_s - 1000}{5}$$

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2) 4 6 5 Brazo S
3) 7 8 9 Brazo S

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$\omega_7 = \omega_5$

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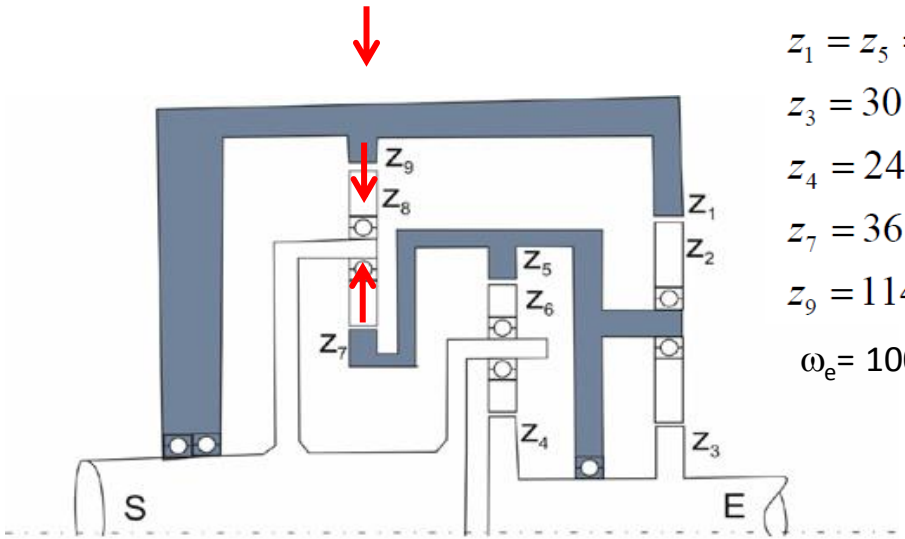

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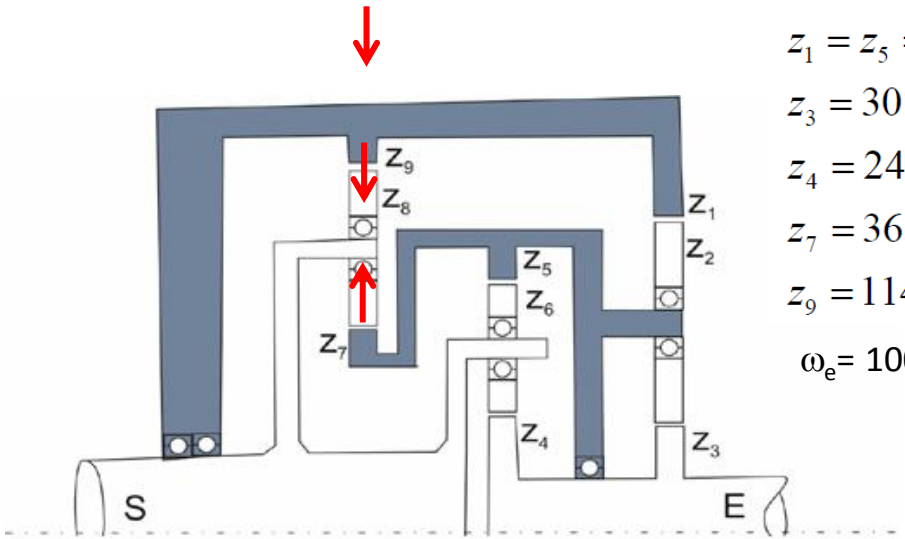
$$\frac{\omega_5 - \omega_s}{\omega_1 - \omega_s} = -\frac{19}{6} \rightarrow \omega_5 = \frac{25\omega_s - 19\omega_1}{6}$$



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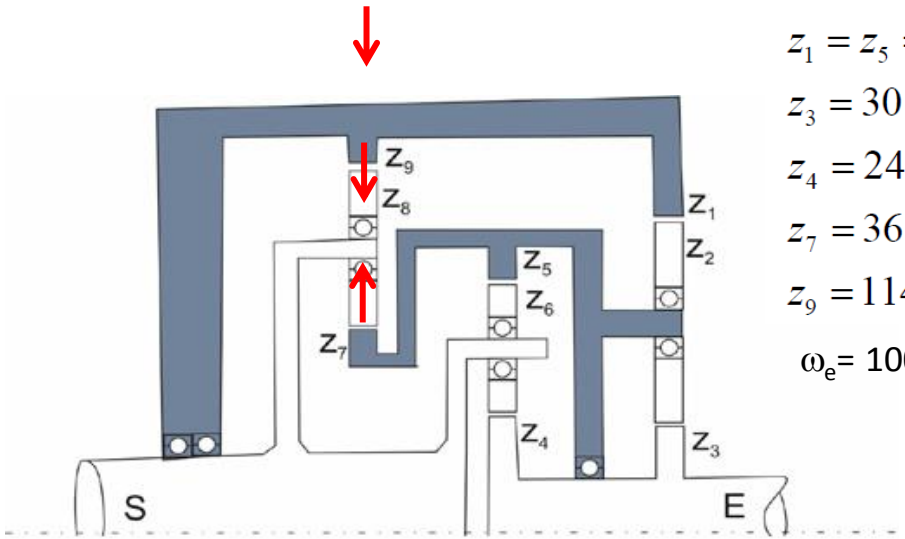

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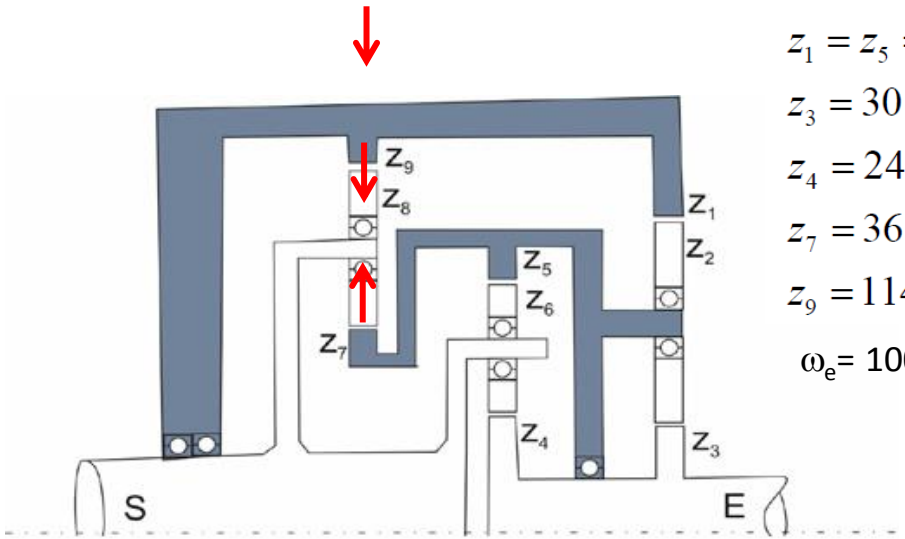

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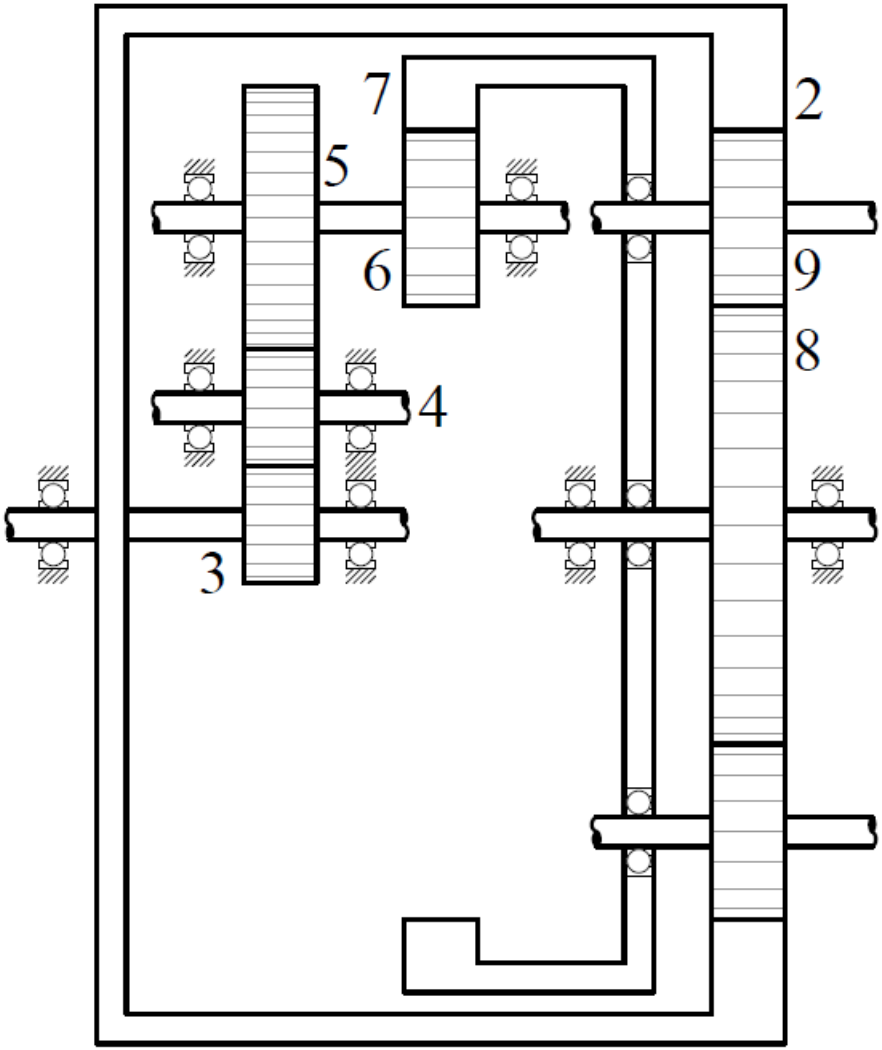
$$\omega_5 = \frac{25\omega_s - 19\omega_1}{6}$$

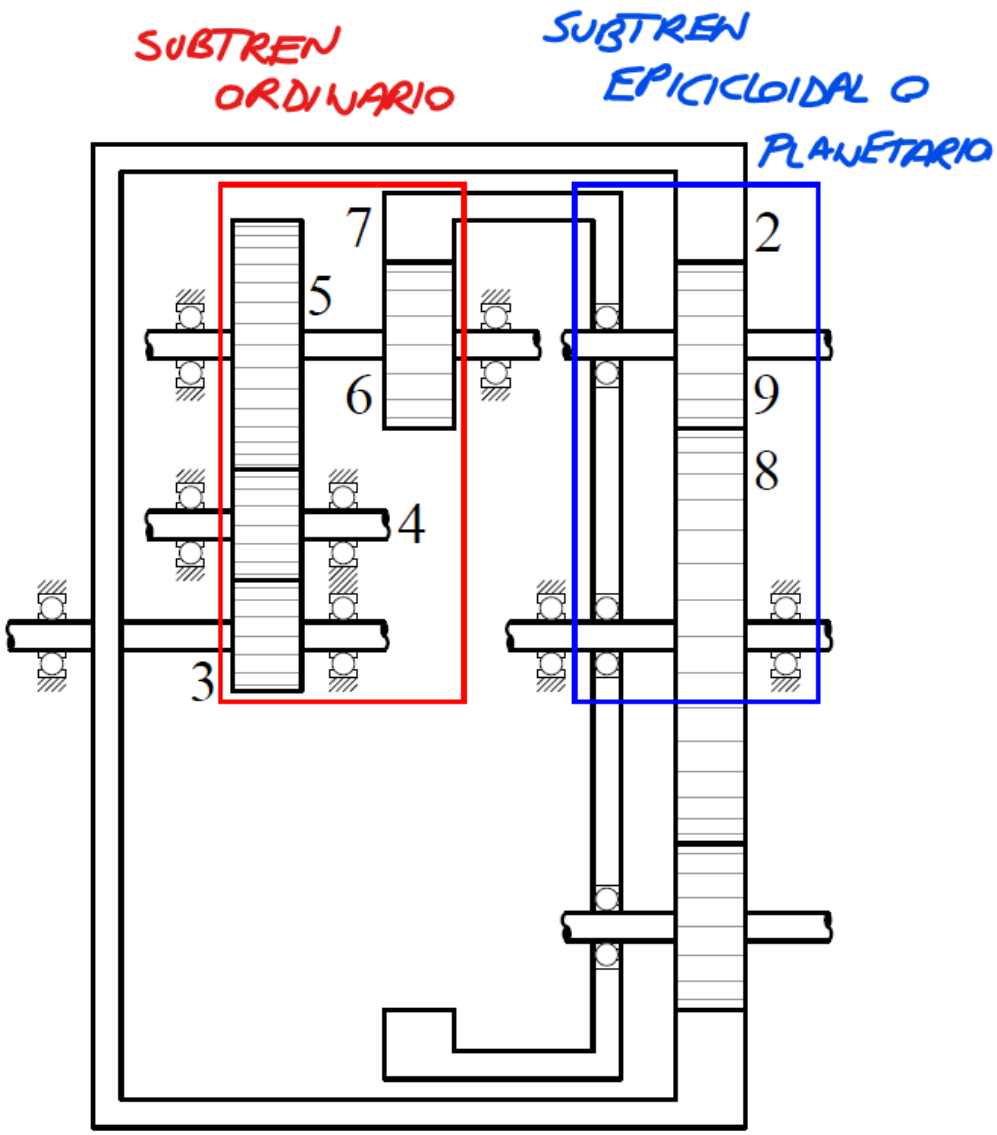
$$\omega_s = \omega_1 = \omega_5 = 1000$$

$$\omega_s = 1000 \text{ rpm (sh)}$$

EJERCICIO 6

$$\omega_8 = f(\omega_3)$$




SUBTREN ORDINARIO
SUBTREN EPICICLOIDAL O PLANETARIO
7
5
6
4
3
2
9
8

ECUACIÓN GENERAL $\Rightarrow$ $\dfrac{w_i - w_R}{w_j - w_R} = \dfrac{w_{iR}}{w_{jR}}$

SUBTREN PLANETARIO O EPICICLOIDAL

2 - 9 - 8 - BRAZO 7

$$\frac{w_2 - w_7}{w_8 - w_7} = \frac{w_{27}}{w_{87}} = \frac{w_{27}}{w_{97}} \cdot \frac{w_{97}}{w_{87}} = \left(\frac{z_9}{z_2}\right)\left(-\frac{z_8}{z_9}\right) = -\frac{z_8}{z_2} \; *$$

SUBTREN ORDINARIO COMPUESTO

3 - 4 - 5 - 6

$$\frac{w_3 - 0}{w_6 - 0} = \frac{w_3}{w_6} = \frac{w_3}{w_4} \cdot \frac{w_4}{w_5} \cdot \frac{w_5}{w_6} = \left(-\frac{z_4}{z_3}\right)\left(-\frac{z_5}{z_4}\right) \cdot 1 = \frac{z_5}{z_3} \; *$$

ECUACIONES DE LIGADURA O DE ENLACE ENTRE TRENES

$w_2 = w_3$ *

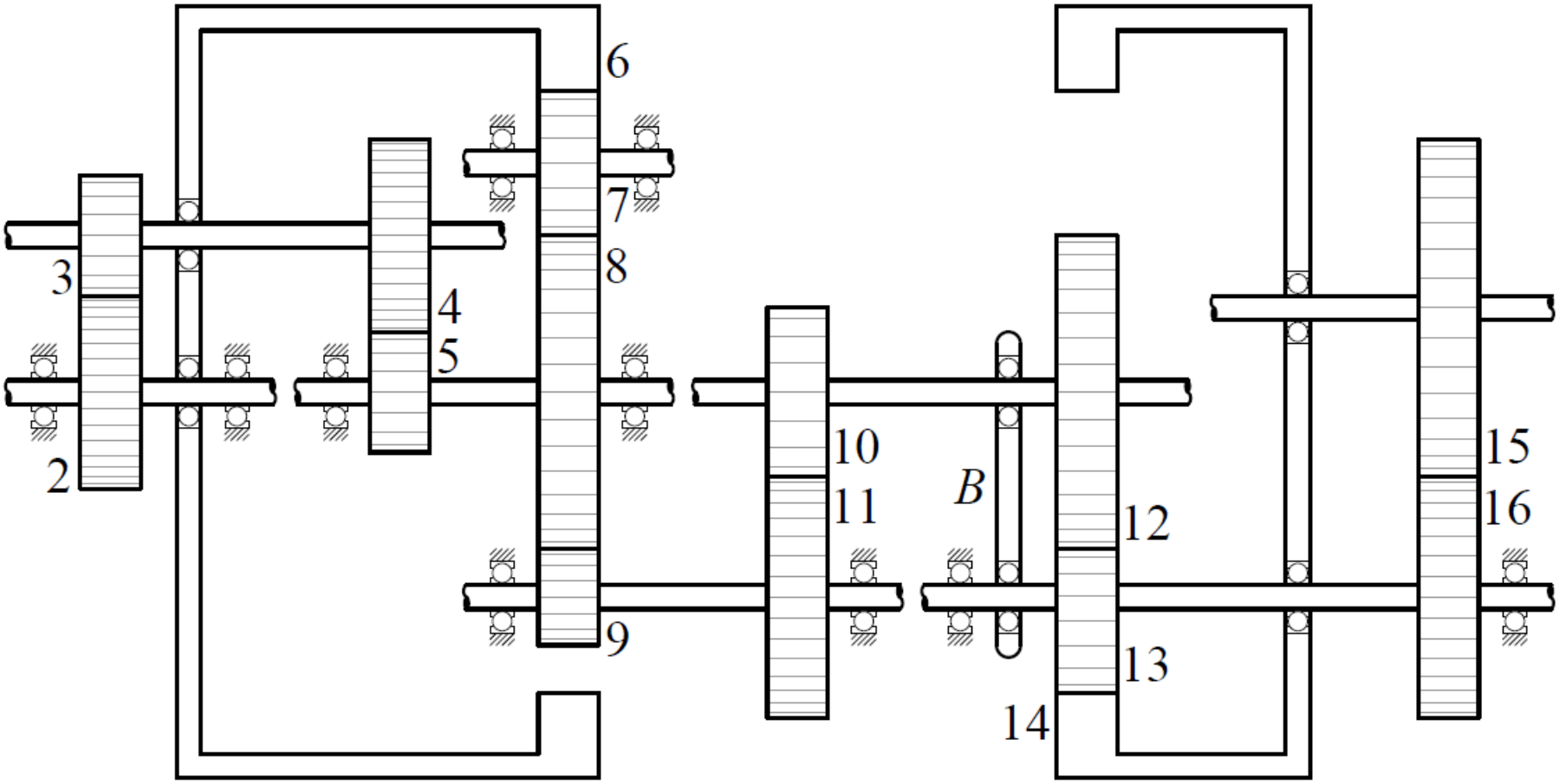
$\dfrac{w_6}{w_7} = -\dfrac{z_7}{z_6}$ *

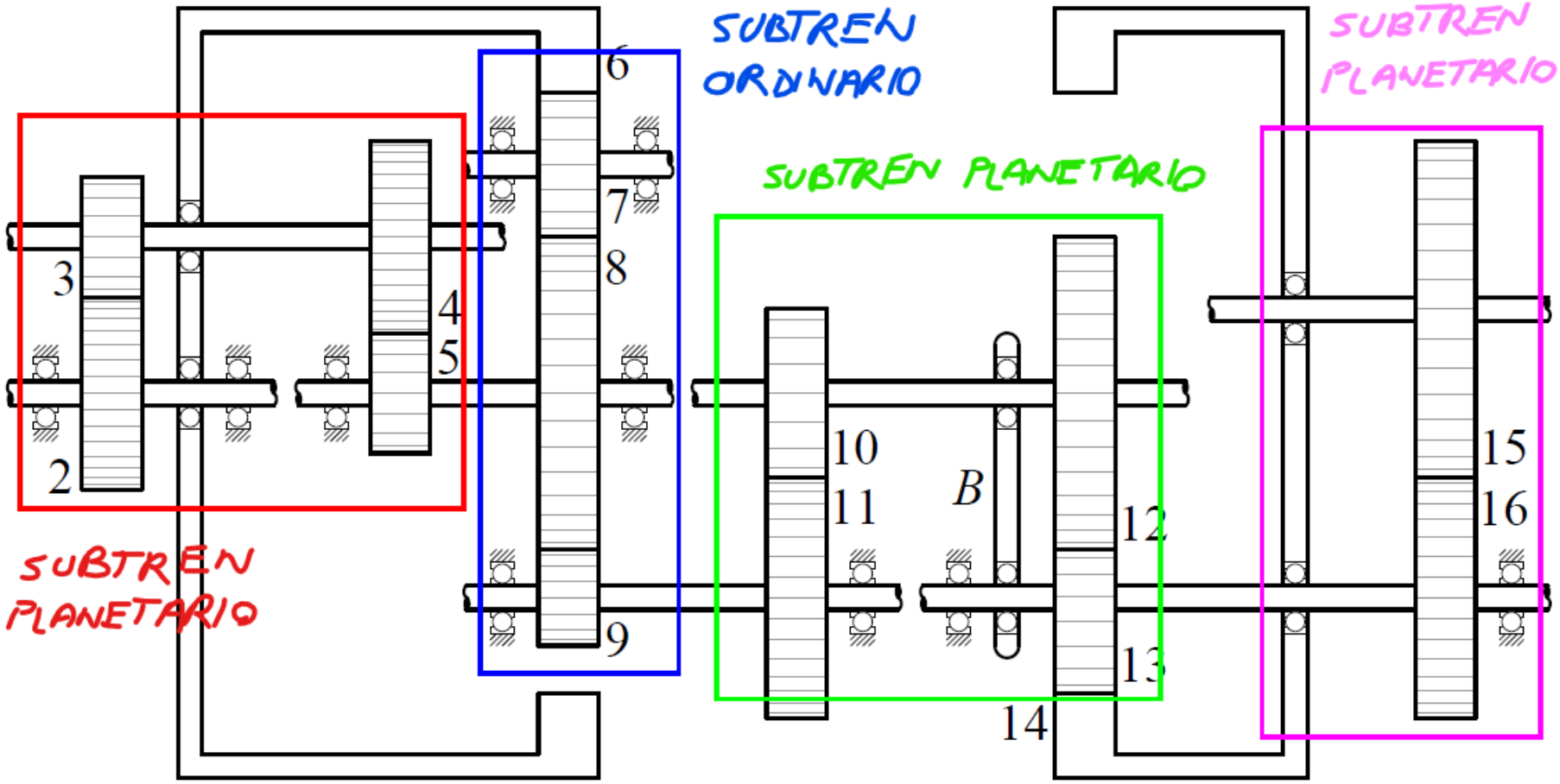
4 ecuaciones * / 4 incógnitas O? } sistema que se puede resolver

$$w_8 = w_3 \left(\frac{z_6 z_3}{z_7 z_5} + \frac{z_9 z_6 z_3}{z_8 z_7 z_5} - \frac{z_9}{z_8}\right)$$

EJERCICIO 7

$$\omega_{16} = f\left(\omega_8, \omega_B\right)$$




SUBTREN ORDINARIO
SUBTREN PLANETARIO
SUBTREN PLANETARIO
SUBTREN PLANETARIO
2
3
4
5
6
7
8
9
10
11
B
12
13
14
15
16

SUBTREN PLANETARIO O EPICICLOIDAL

2 - 3 - 4 - 5 - BRAZO 6

$$\frac{\omega_2 - \omega_6}{\omega_5 - \omega_6} = \frac{\omega_{26}}{\omega_{56}} = \frac{\omega_{26}}{\omega_{36}} \cdot \frac{\omega_{36}}{\omega_{46}} \cdot \frac{\omega_{46}}{\omega_{56}} = \left(-\frac{z_3}{z_2}\right) \cdot 1 \cdot \left(-\frac{z_5}{z_4}\right) = \frac{z_3 z_5}{z_2 z_4} \; *$$

SUBTREN ORDINARIO

6 - 7 - 8 - 9

$$\frac{\omega_6}{\omega_9} = \frac{\omega_6}{\omega_7} \cdot \frac{\omega_7}{\omega_8} \cdot \frac{\omega_8}{\omega_9} = \left(\frac{z_7}{z_6}\right) \cdot \left(-\frac{z_8}{z_7}\right) \cdot \left(-\frac{z_9}{z_8}\right) = \frac{z_9}{z_6} \; *$$

SUBTREN PLANETARIO

11 - 10 - 12 - 13 - BRAZO 8

$$\frac{\omega_{11} - \omega_8}{\omega_{13} - \omega_8} = \frac{\omega_{118}}{\omega_{138}} = \frac{\omega_{118}}{\omega_{108}} \cdot \frac{\omega_{108}}{\omega_{128}} \cdot \frac{\omega_{128}}{\omega_{138}} = \left(-\frac{z_{10}}{z_{11}}\right) \cdot 1 \cdot \left(-\frac{z_{13}}{z_{12}}\right) = \frac{z_{10} z_{13}}{z_{11} z_{12}} \; *$$

SUBTREN PLANETARIO

15 - 16 - BRAZO 14

$$\frac{\omega_{16} - \omega_{14}}{\omega_{15} - \omega_{14}} = \frac{\omega_{1614}}{\omega_{1514}} = -\frac{z_{15}}{z_{16}} \; *$$

ECUACIONES DE ENLACE ENTRE SUBTRENES

$$\frac{w_6}{w_7} = -\frac{z_7}{z_6} \quad *$$

$$w_9 = w_{11} \quad *$$

$$\frac{w_{13} - w_B}{w_{14} - w_B} = \frac{w_{BB}}{w_{14B}} = \frac{z_{14}}{z_{13}} \quad *$$

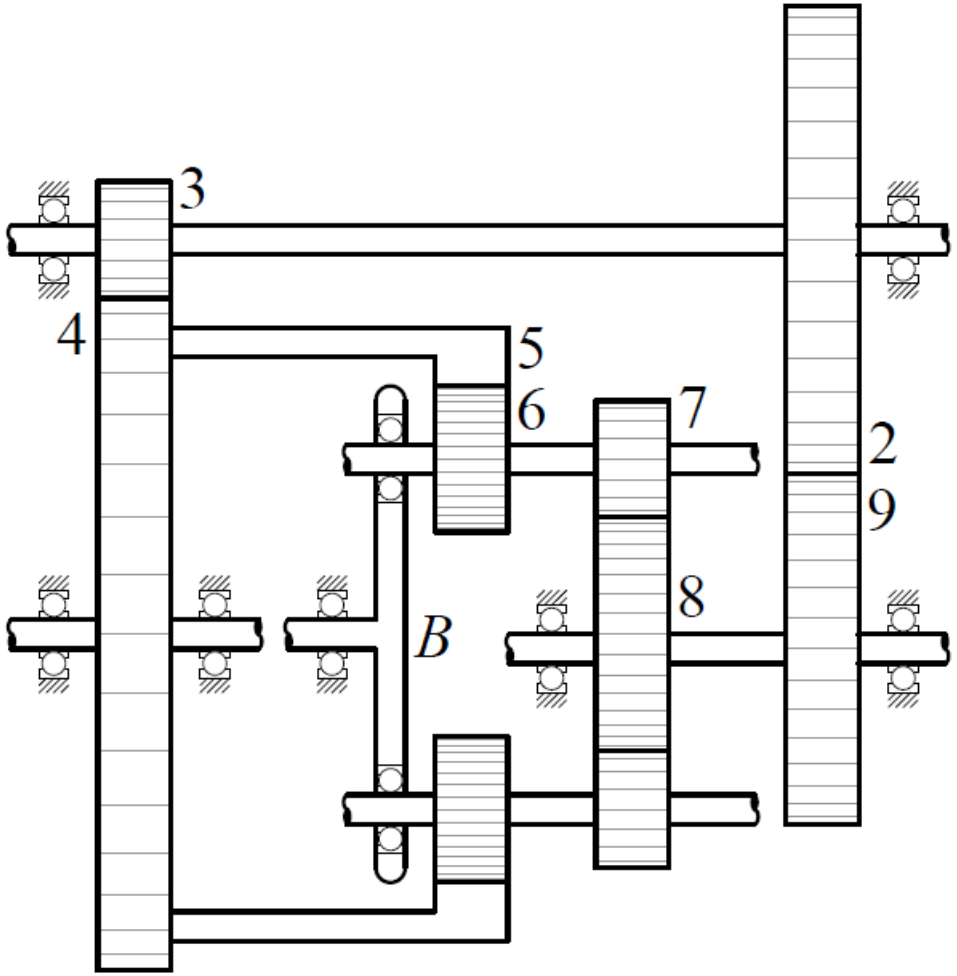
$$w_{13} = w_{1C} \quad *$$

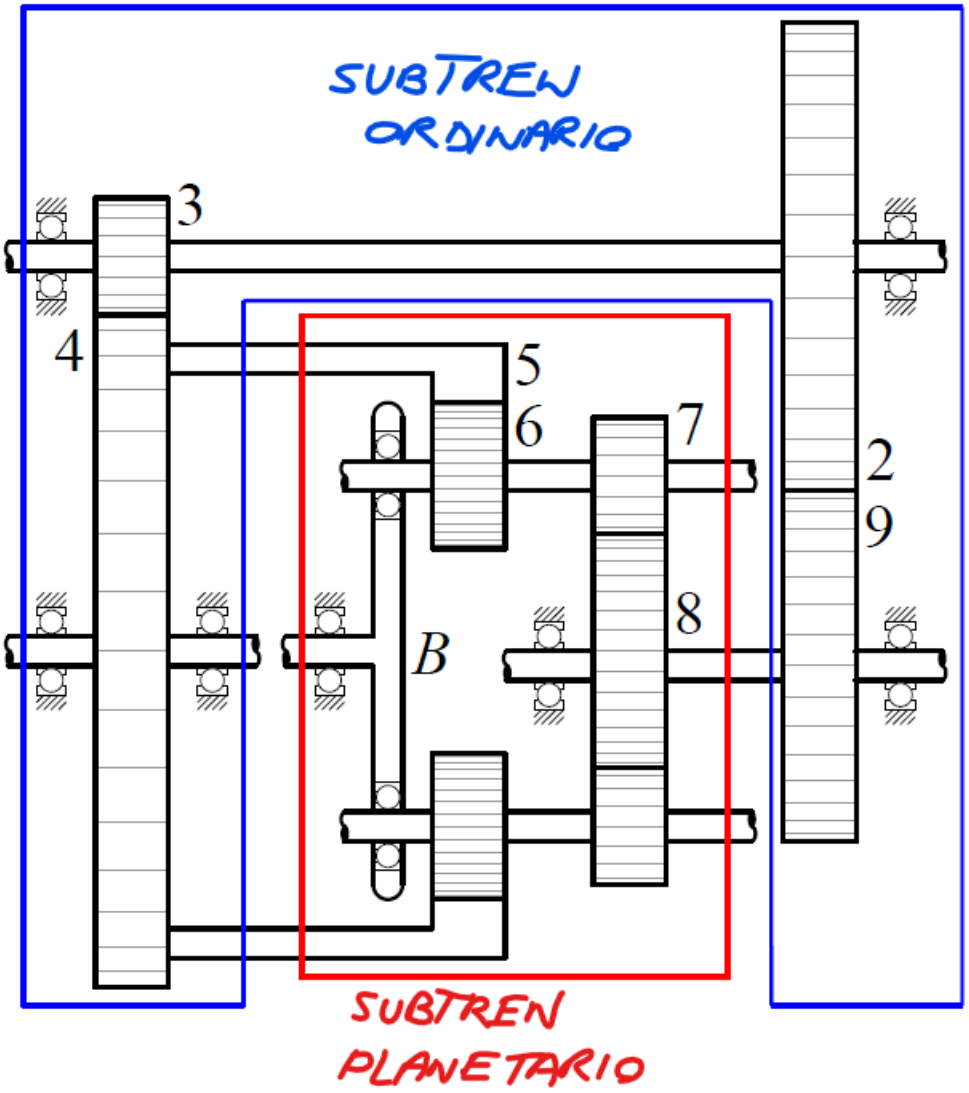
$$w_5 = w_8 \quad *$$

9 ecuaciones
9 incógnitas
} RESOLUBLE

EJERCICIO 8

$$\omega_B = f(\omega_9)$$




SUBTREN
ORDINARIO
3
4
5
6
7
2
9
8
B
SUBTREN
PLANETARIO

ω_9 ✓ ω_B ?

TREN ORDINARIO

4 - 3 - 2 - 9

$$\frac{\omega_4 - 0}{\omega_9 - 0} = \frac{\omega_4}{\omega_9} = \frac{\omega_4}{\omega_3} \cdot \frac{\omega_3}{\omega_2} \cdot \frac{\omega_2}{\omega_9} = \left(-\frac{z_3}{z_4}\right) \cdot 1 \cdot \left(-\frac{z_9}{z_2}\right) = \frac{z_3 z_9}{z_4 z_2} \ *$$

TREN PLANETARIO O EPICICLOIDAL

5 - 6 - 7 - 8 - BRAZO B

$$\frac{\omega_5 - \omega_B}{\omega_8 - \omega_B} = \frac{\omega_{5B}}{\omega_{8B}} = \frac{\omega_{5B}}{\omega_{6B}} \cdot \frac{\omega_{6B}}{\omega_{7B}} \cdot \frac{\omega_{7B}}{\omega_{8B}} = \left(\frac{z_6}{z_5}\right) \cdot 1 \cdot \left(-\frac{z_8}{z_7}\right) = -\frac{z_6 z_8}{z_5 z_7} \ *$$

ECUACIONES DE LIGADURA O DE ENLACE

$\omega_4 = \omega_5$ *

$\omega_8 = \omega_9$ *

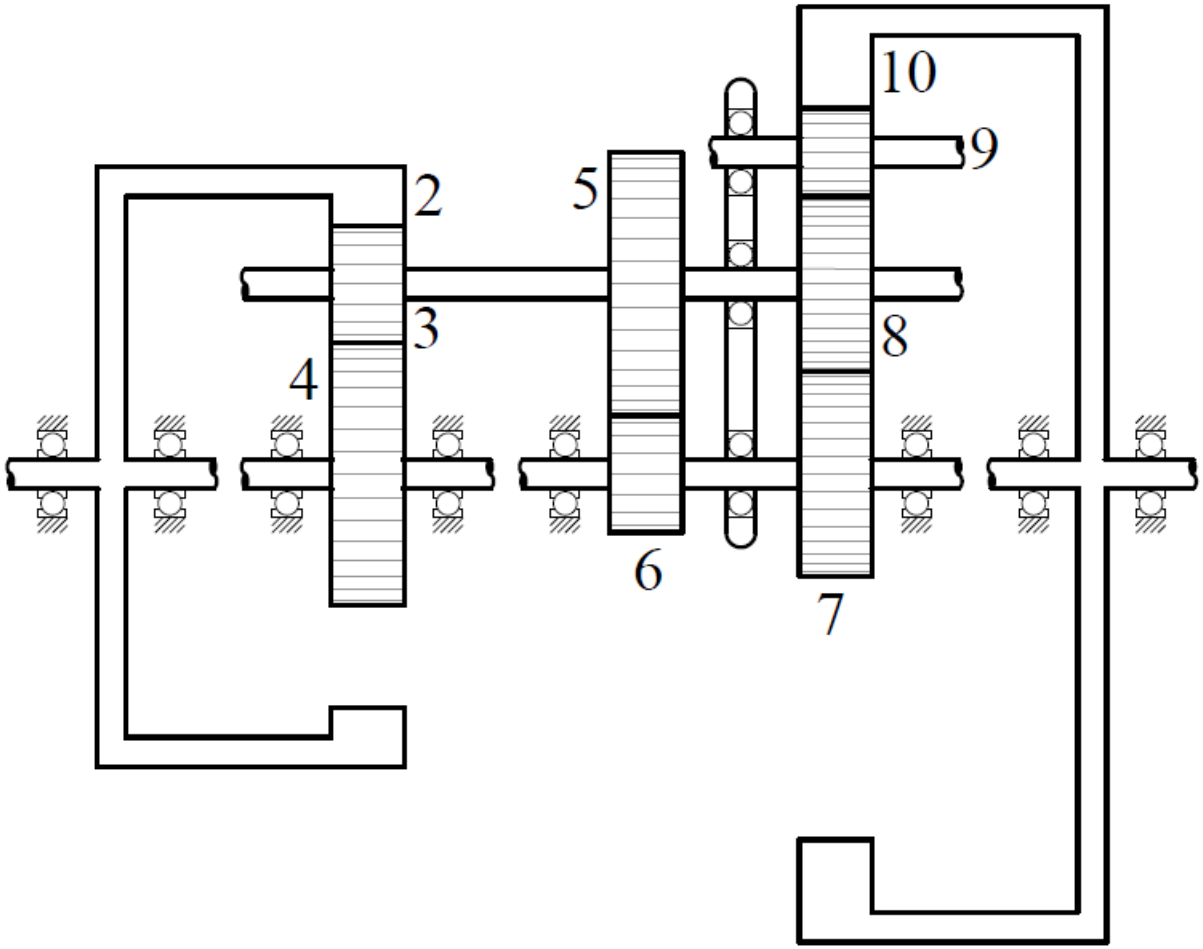
4 ecuaciones y 4 incógnitas ⇒ SISTEMA RESOLUBLE

$$\omega_B = \omega_9 \cdot \frac{\left(\frac{z_6 z_8}{z_5 z_7} - \frac{z_9}{z_2}\right)}{\left(1 + \frac{z_6 z_8}{z_5 z_7}\right)}$$

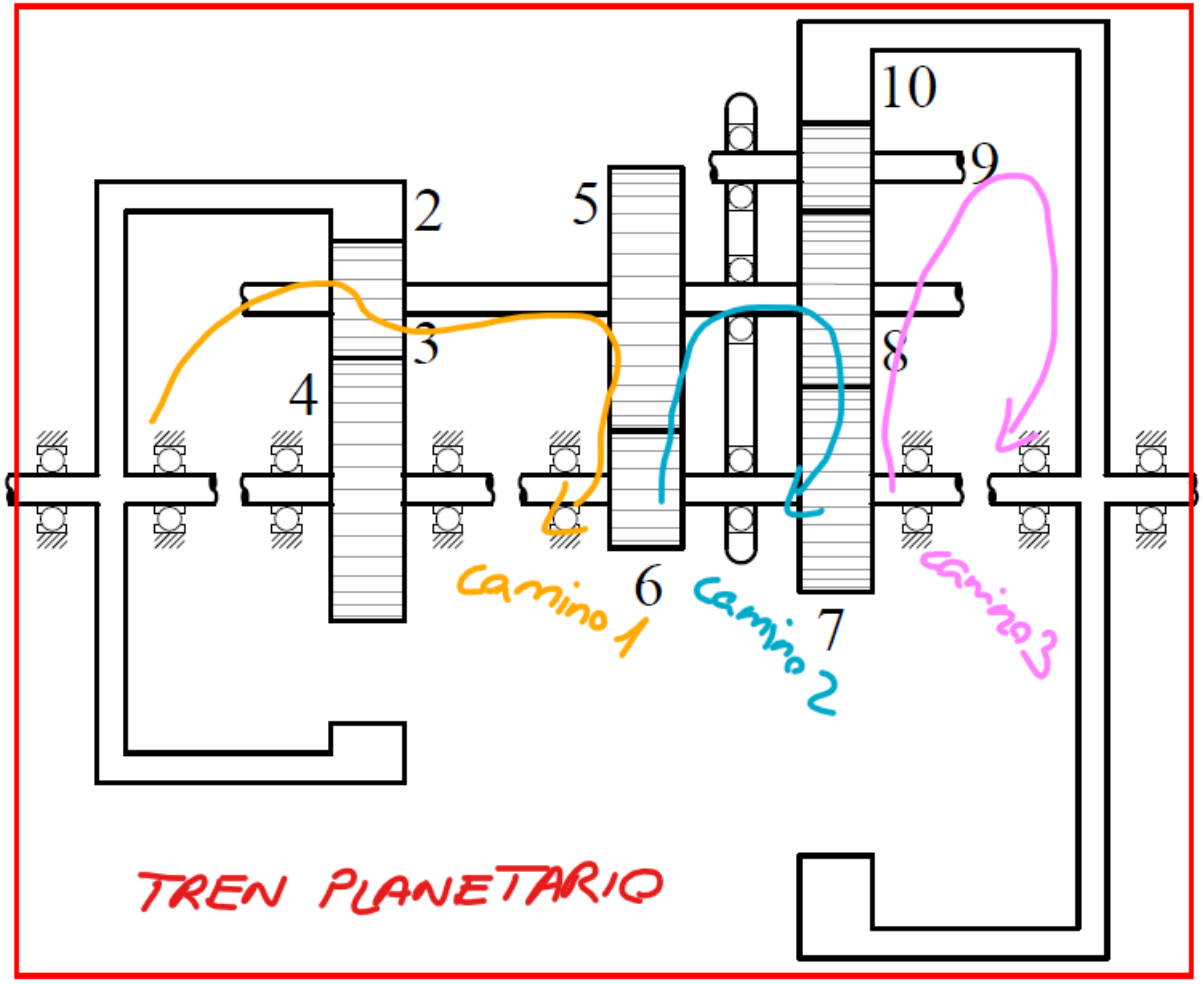
FACTOR DEL TREN

EJERCICIO 9

$$\omega_9 = f\left(\omega_2, \omega_7\right)$$



Este tren planetario tiene un único brazo, dos soles (ruedas dentadas 4, 6 y 8), dos coronas (ruedas dentadas 10 y 2) y varios satélites. En este tren planetario hay tres caminos para recorrer la totalidad de los eslabones o engranajes que lo componen. Los tres caminos se pueden ver como tres subtrenes planetarios y cada subtren tendrá su ecuación de movimiento.



w_2 ✓ w_7 ✓ ¿ w_9 ?

Camino 1 · SUBTREN PLANETARIO 2-3-5-6-BRAZO b

✓ $\frac{w_2 - w_b}{?\ w_6 - w_b} = \frac{w_{2b}}{w_{6b}} = \frac{w_{2b}}{w_{3b}} \cdot \frac{w_{3b}}{w_{5b}} \cdot \frac{w_{5b}}{w_{6b}} = \left(\frac{z_3}{z_2}\right) \cdot 1 \cdot \left(-\frac{z_6}{z_5}\right) = -\frac{z_3 z_6}{z_2 z_5}$ *

Camino 2 · SUBTREN PLANETARIO 6-5-8-7-BRAZO b

✓ $\frac{w_6 - w_b}{w_7 - w_b} = \frac{w_{6b}}{w_{7b}} = \frac{w_{6b}}{w_{5b}} \cdot \frac{w_{5b}}{w_{8b}} \cdot \frac{w_{8b}}{w_{7b}} = \left(-\frac{z_5}{z_6}\right) \cdot 1 \cdot \left(-\frac{z_7}{z_8}\right) = \frac{z_5 z_7}{z_6 z_8}$ *

Camino 3 · SUBTREN PLANETARIO 7-8-9-10-BRAZO b

? $\frac{w_7 - w_b}{w_{10} - w_b} = \frac{w_{7b}}{w_{10b}} = \frac{w_{7b}}{w_{8b}} \cdot \frac{w_{8b}}{w_{9b}} \cdot \frac{w_{9b}}{w_{10b}} = \left(-\frac{z_8}{z_7}\right) \cdot \left(-\frac{z_7}{z_8}\right) \left(\frac{z_{10}}{z_9}\right) = \frac{z_{10}}{z_7}$ *

ECUACIONES DE LIGADURA

$w_6 = w_7$ *

4 ecuaciones y 4 incógnitas ⇒ RESOLUBLE

$$\left.\begin{aligned} \frac{w_2 - w_b}{w_3 - w_b} &= -\frac{z_3 z_6}{z_2 z_5} \\ \frac{w_7 - w_b}{w_{10} - w_b} &= \frac{z_{10}}{z_7} \end{aligned}\right\}$$ 2 ecuaciones y 2 incógnitas: w_b y w_{10}

Se pide $w_9 = f(w_2, w_7) \Rightarrow \frac{w_9 - w_b}{w_3 - w_b} = -\frac{z_7}{z_9} \rightsquigarrow \boxed{w_9}$